Computational Challenges in Climate Modeling

Invited Session, American Physical Society New Orleans, 13 March 2008

Predicting Climate Change:
Uncertainties and prospects for surmounting them

Michael Ghil

ENS, Paris, and UCLA

M. D. Chekroun, ENS (Paris); D. Kondrashov, UCLA;
E. Simonnet, INLN (Nice); and I. Zaliapin, U. Nevada, Reno

Pls. see these sites for further info.

http://www.atmos.ucla.edu/tcd/ (TCD and IPCC)

http://www.environnement.ens.fr/

Motivation

- The ***climate system*** is highly ***nonlinear and*** quite ***complex***.
- Its ***major components*** — the atmosphere, oceans, ice sheets — ***flow*** on many time and space scales.
- Its ***predictive understanding*** has to rely on the system's physical, chemical and biological modeling,
  but also on the mathematical analysis of the models
  thus obtained.
- The ***hierarchical modeling*** approach allows one to
  give proper weight to the understanding provided by the
  models *vs*. their realism, respectively.
- This approach facilitates the evaluation of ***forecasts (pognostications?)*** based on these models.
- Back-and-forth between ***"toy"*** (conceptual) and ***detailed*** ("realistic") ***models***, and between ***models*** and ***data***.

Outline

- The IPCC process: results (!!) and questions (???)
- Natural climate variability: source of uncertainties
  - sensitivity to initial state => error growth
  - sensitivity to model formulation =>
    change in means & variances – see below!
- Uncertainties and how to fix them
  - structural instability – ENSO-FDE model
  - random dynamical systems – toy models
- Conclusions and references
  - computational cost?

(!!) Nobel Peace Prize!!; (???) So what's next???

Global warming and its socio-economic impacts

Temperatures rise:

- What about impacts?
- How to adapt?

The answer, my friend, is blowing in the wind, *i.e.*, it depends on the accuracy and reliability of the forecast …

Source : IPCC (2007), *AR4, WGI, SPM*

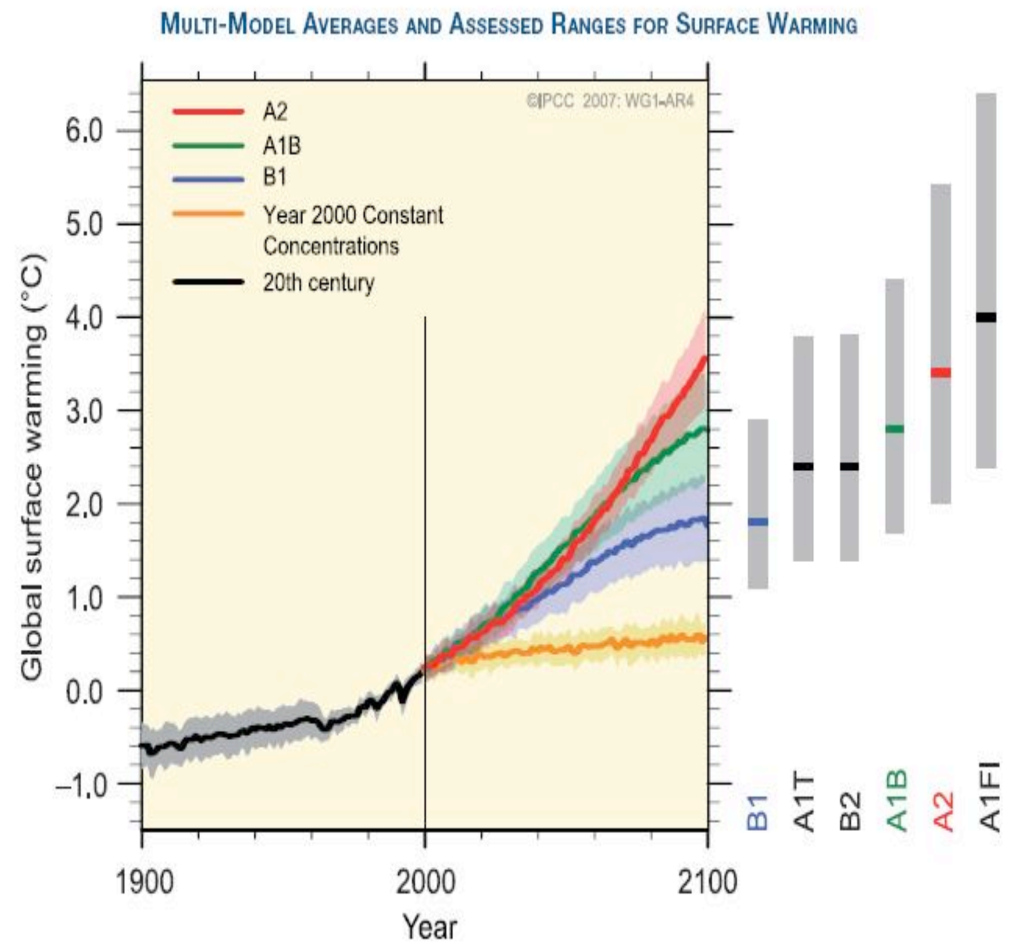


Figure SPM.5. *Solid lines are multi-model global averages of surface warming (relative to 1980–1999) for the scenarios A2, A1B and B1, shown as continuations of the 20th century simulations. Shading denotes the ±1 standard deviation range of individual model annual averages. The orange line is for the experiment where concentrations were held constant at year 2000 values. The grey bars at right indicate the best estimate (solid line within each bar) and the* **likely** *range assessed for the six SRES marker scenarios. The assessment of the best estimate and* **likely** *ranges in the grey bars includes the AOGCMs in the left part of the figure, as well as results from a hierarchy of independent models and observational constraints. {Figures 10.4 and 10.29}*

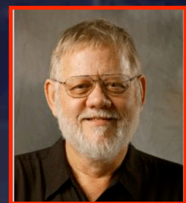
A Differential Delay Model of ENSO Variability:
Parametric Instability and Distribution of Extremes

Michael Ghil

Ecole Normale Supérieure, Paris,
Univ. of California, Los Angeles

Ilya Zaliapin

Univ. of Nevada, Reno



Skip Thompson

Radford Univ., Virginia

Outline

- Motivation – topic and model
- ENSO and model formulation
- Results
  - Theoretical and numerical results
  - Interannual, interdecadal and intraseasonal variability
  - Smooth and sharp transitions in behavior
  - Spontaneous changes in mean and extremes
  - Multiplicity of solutions
  - Phase locking and Devil's staircase
- Concluding remarks

Motivation – choice of topic

- Climate models -- the most sophisticated models of natural phenomena.
- Still, the range of uncertainty in responses to CO_2 doubling is not decreasing.
- Can this be a matter of intrinsic sensitivity to model parameters and parameterizations, similar to but distinct from sensitivity to initial data?
- Dynamical systems theory has, so far, interpreted model robustness in terms of *structural stability*; it turns out that this property is not generic.
- We explore the structurally unstable behavior of a toy model of ENSO variability, the interplay between forcing and internal variability, as well as spontaneous changes in mean and extremes.

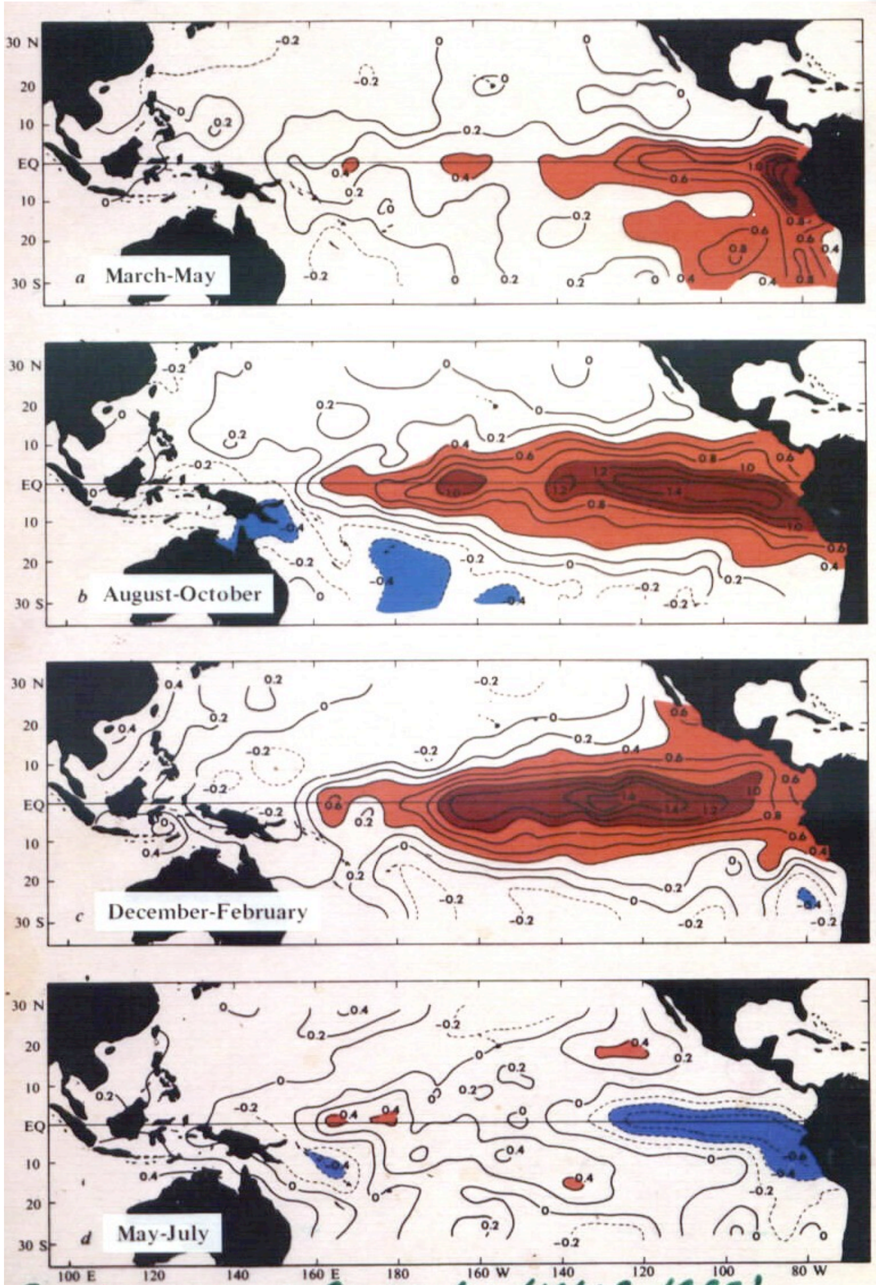
Motivation – choice of "toy model"

Differential Delay Equations (DDE) offer an effective modeling language as they combine simplicity of formulation with rich behavior…

To gain some intuition, compare

ODE	DDE
$f'(t) = \alpha f(t), \ \alpha > 0$	$f'(t) = \alpha f(t-\tau), \ \alpha > 0$
The only solution is $$f(t) = e^{\alpha t}$$ *i.e.*, exponential growth (or decay, for $\alpha < 0$)	The general solution is given by $$f(t) = Ce^{rt} + \sum_{k=1}^{\infty} e^{p_k t}\left(A_k \cos(q_k t) + B_k \sin(q_k t)\right)$$ A_k, B_k, C – arbitrary r - the only real root of $xe^{x\tau} = \alpha$ $(p_k \pm i q_k)$ - complex roots of $xe^{x\tau} = \alpha$ In particular, oscillatory solutions do exist.

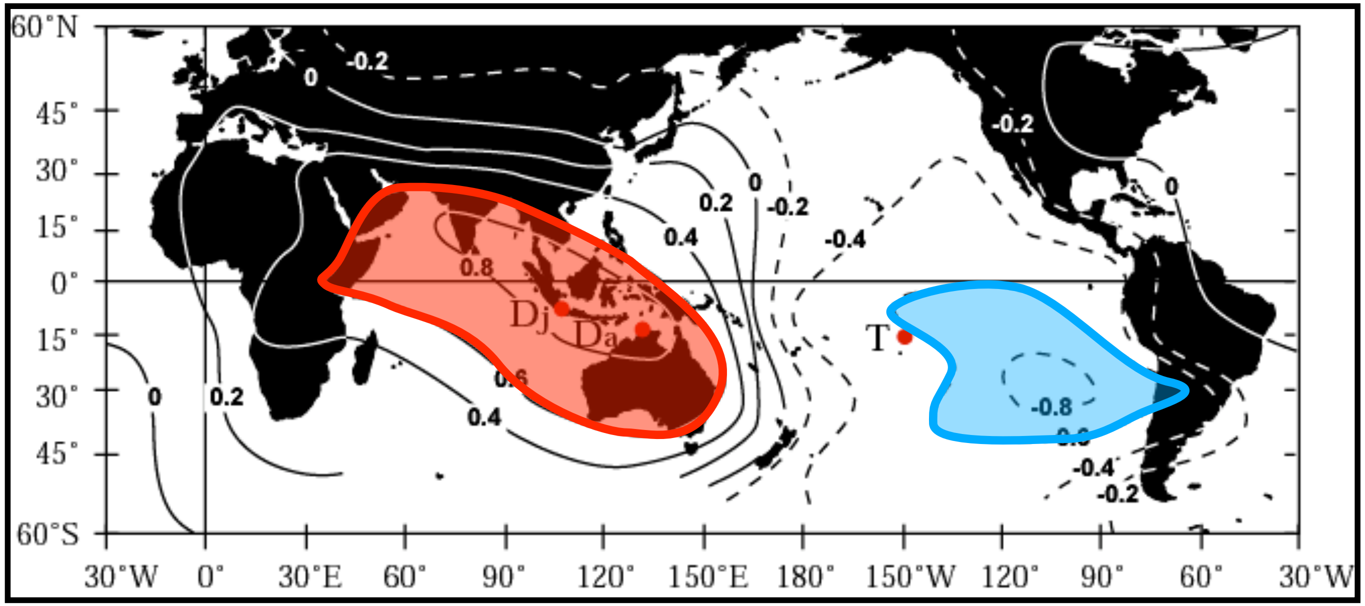
Spatio-temporal evolution of ENSO episode



Rasmusson & Carpenter (MWR, 1982)

Scalar time series that capture ENSO variability

The large-scale ***Southern Oscillation (SO) pattern*** associated with ***El Niño (EN),*** as originally seen in surface pressures



Neelin (2006) *Climate Modeling and Climate Change*, after Berlage (1957)

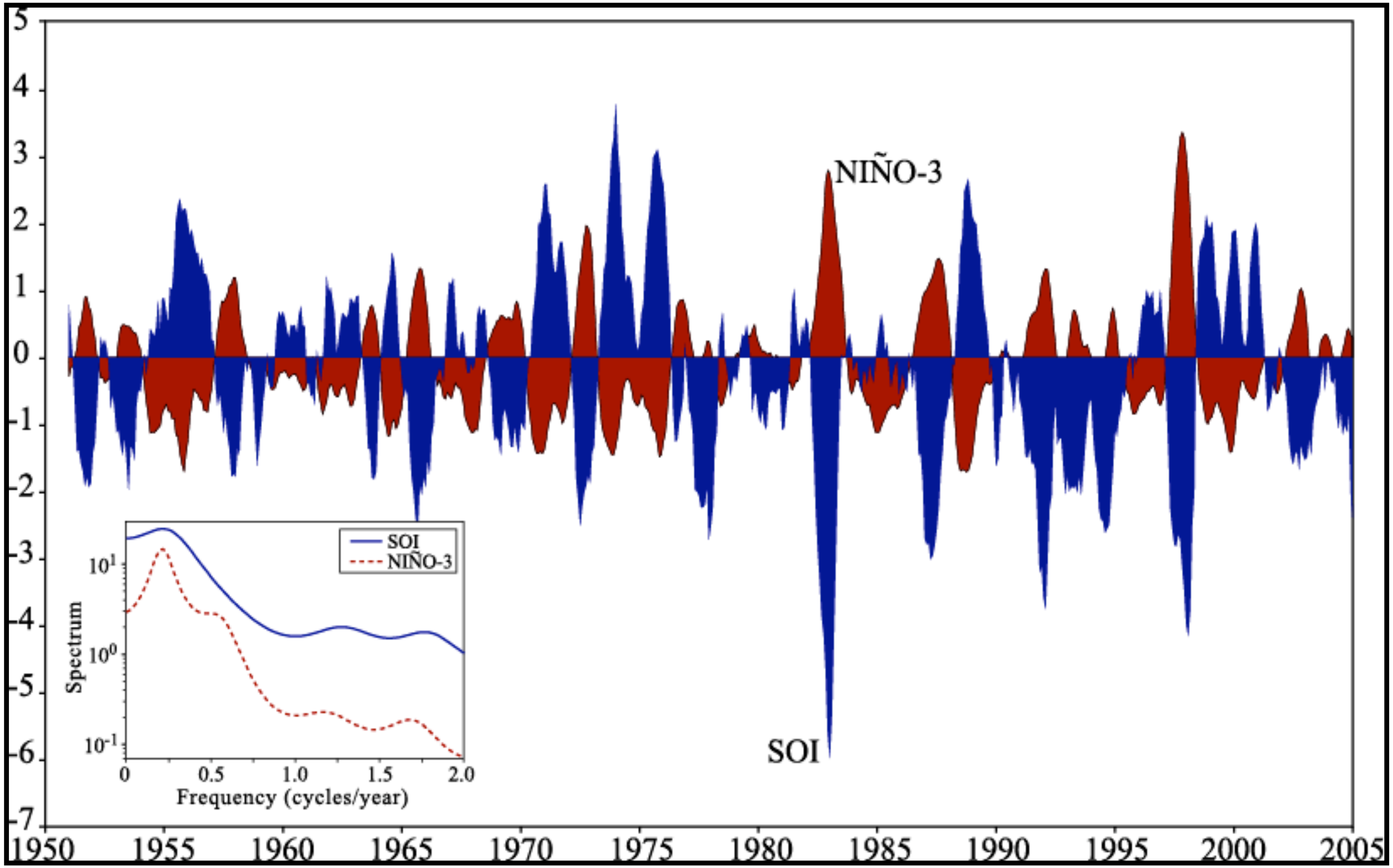
Southern Oscillation:

The seesaw of sea-level pressures p_s between the two branches of the Walker circulation

Southern Oscillation Index (SOI) = normalized difference between p_s at Tahiti (T) and p_s at Darwin (Da)

Scalar time series that capture ENSO variability

Time series of ***atmospheric pressure***
and ***sea surface temperature*** (SST) indices



Data courtesy of NCEP's Climate Prediction Center
Neelin (2006) *Climate Modeling and Climate Change*

Model formulation

$$\frac{d}{dt}h(t) = -\tanh\left[\kappa h(t-\tau)\right] + b\cos(2\pi t)$$

Thermocline depth deviations from the annual mean in the eastern Pacific

Wind-forced ocean waves (E'ward Kelvin, W'ward Rossby)

Strength of the atmosphere-ocean coupling

Delay due to finite wave velocity

Seasonal-cycle forcing

Model parameters (cont'd)

$$\frac{d}{dt}h(t) = -\tanh\left[\kappa h(t-\tau)\right] + b\cos(2\pi\omega t)$$

The seasonal-cycle forcing has the period P_0:

$$P_0 = (\omega)^{-1} = 1 \text{ yr},$$

and we consider the following parameter ranges:

$$0 \leq \tau \leq 2 \text{ [yr]}$$

$$0 < \kappa < \infty$$

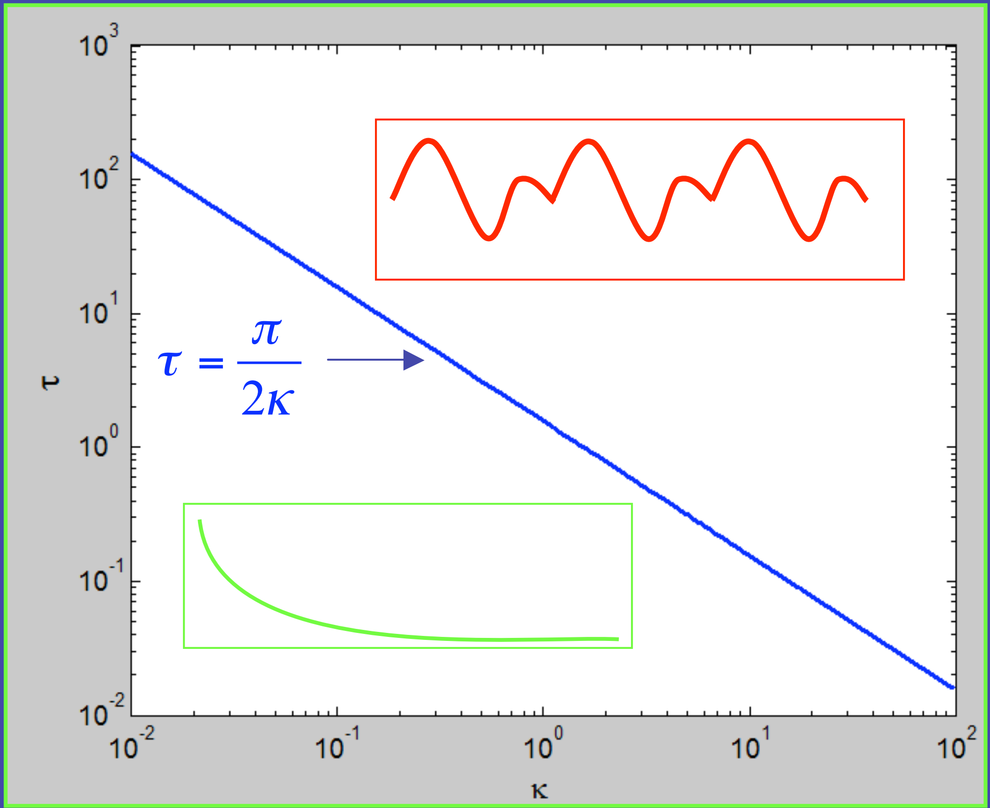
$$0 \leq b \leq \infty$$

The initial data for our DDE are given by the constant history (warm event):

$$h(t) = 1, \; -\tau \leq t < 0$$

Model: general results

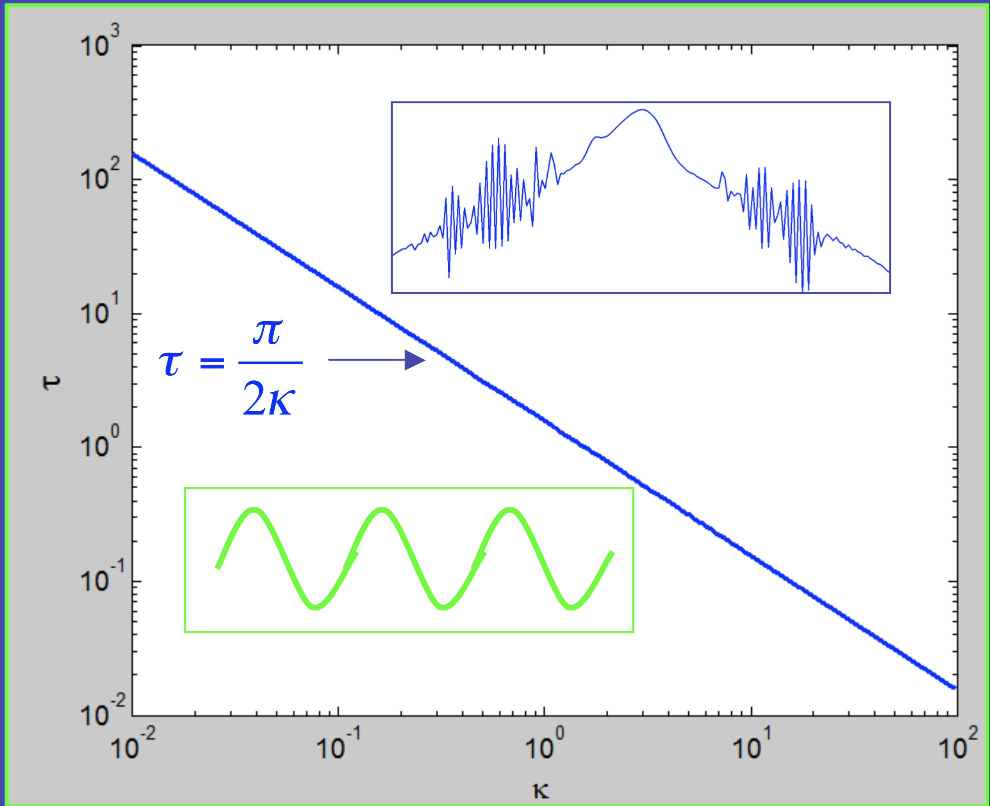
With no seasonal forcing we have $$\frac{d}{dt}h(t) = -A\left[h(t-\tau)\right]$$



- For "large" delays, the solution is asymptotically periodic, with period 4τ
- For "small" delays, the solution is asymptotically zero, as it is for no delay (ODE case)

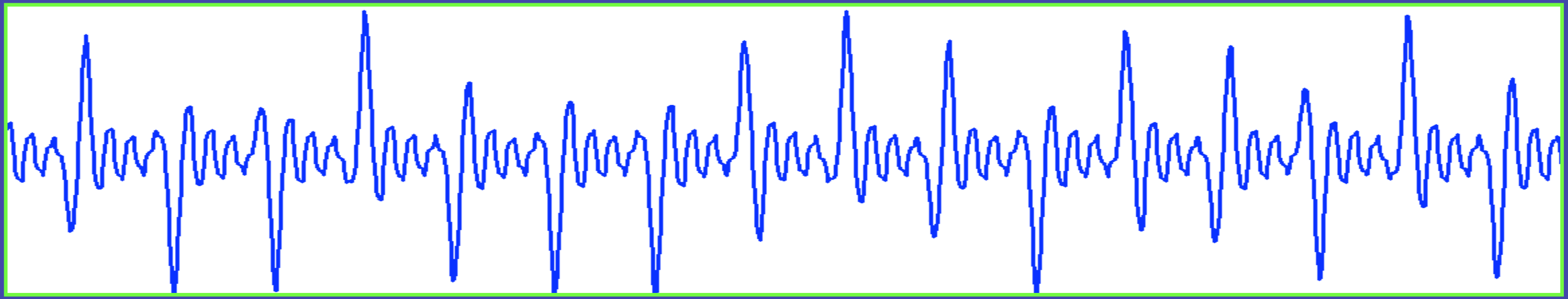
Model: general results (cont'd)

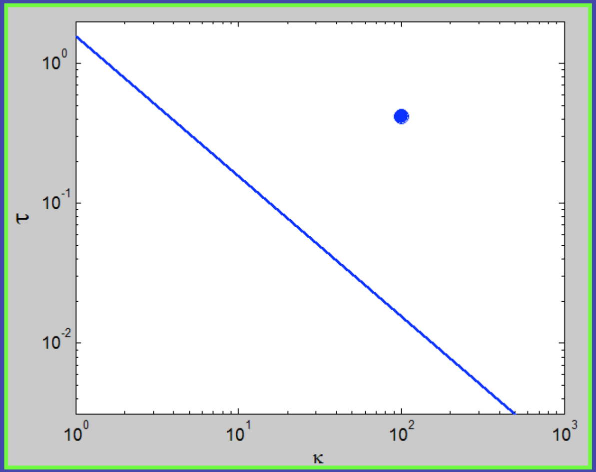
… accordingly, for $$\frac{d}{dt}h(t) = -A\left[h(t-\tau)\right] + b\cos(2\pi\omega t)$$



- For "large" delays, there are ***nonlinear interactions*** between periodic solutions with periods 4τ and 1
- For "small" delays, the solution is asymptotically periodic with period 1, as for the no-delay (ODE) case

Noteworthy scenarios (2)



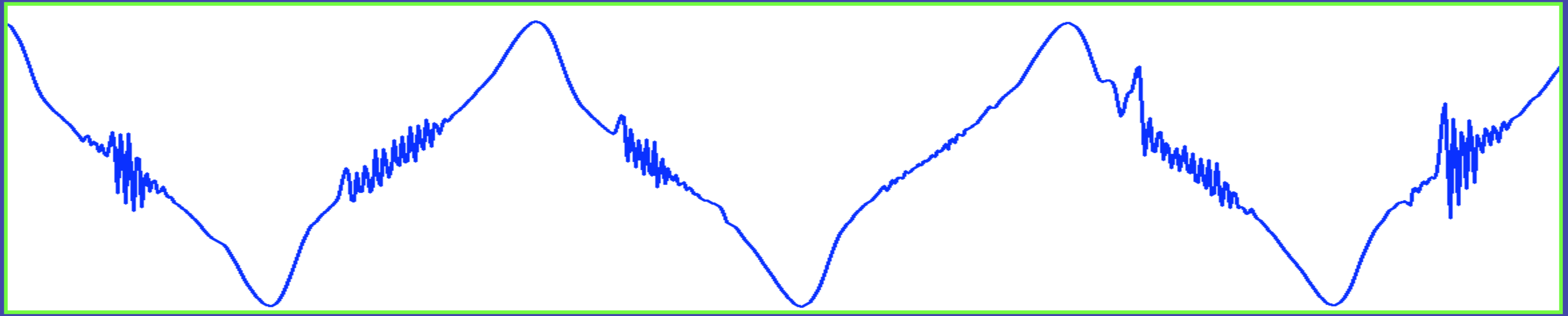


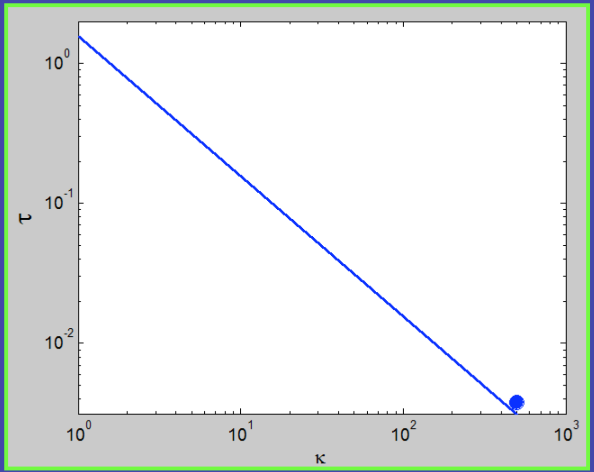
$b = 1, \ \kappa = 100, \ \tau = 0.42$

"High-*h*" season with period of about 4 yr; notice the random heights of high seasons

N.B. Rough equivalent of El Niño in this "toy model" (little upwelling near coast)

Noteworthy scenarios (3)



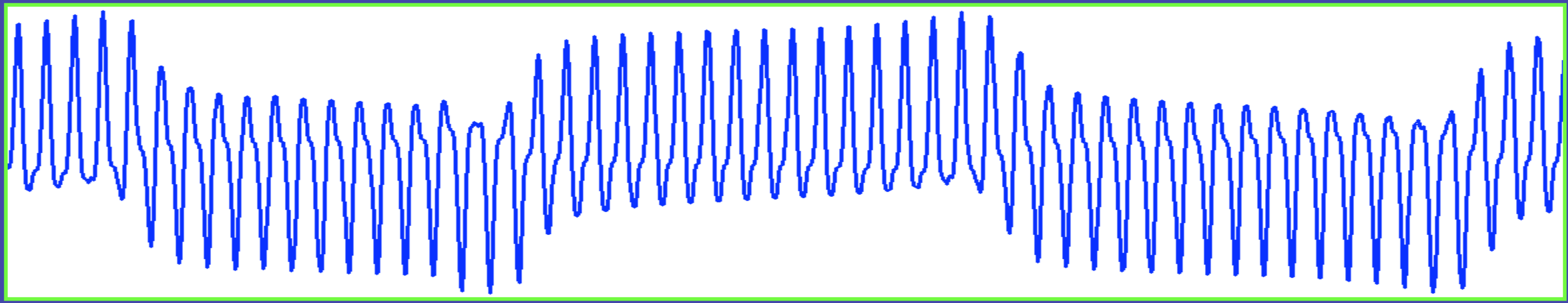


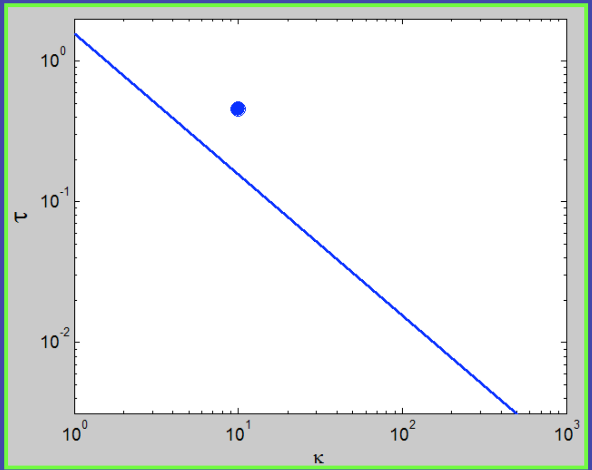
$b = 1,\ \kappa = 500,\ \tau = 0.0038$

Bursts of intraseasonal oscillations (*) of random amplitude

(*) Madden-Julian oscillations, westerly-wind bursts?

Noteworthy scenarios (4)





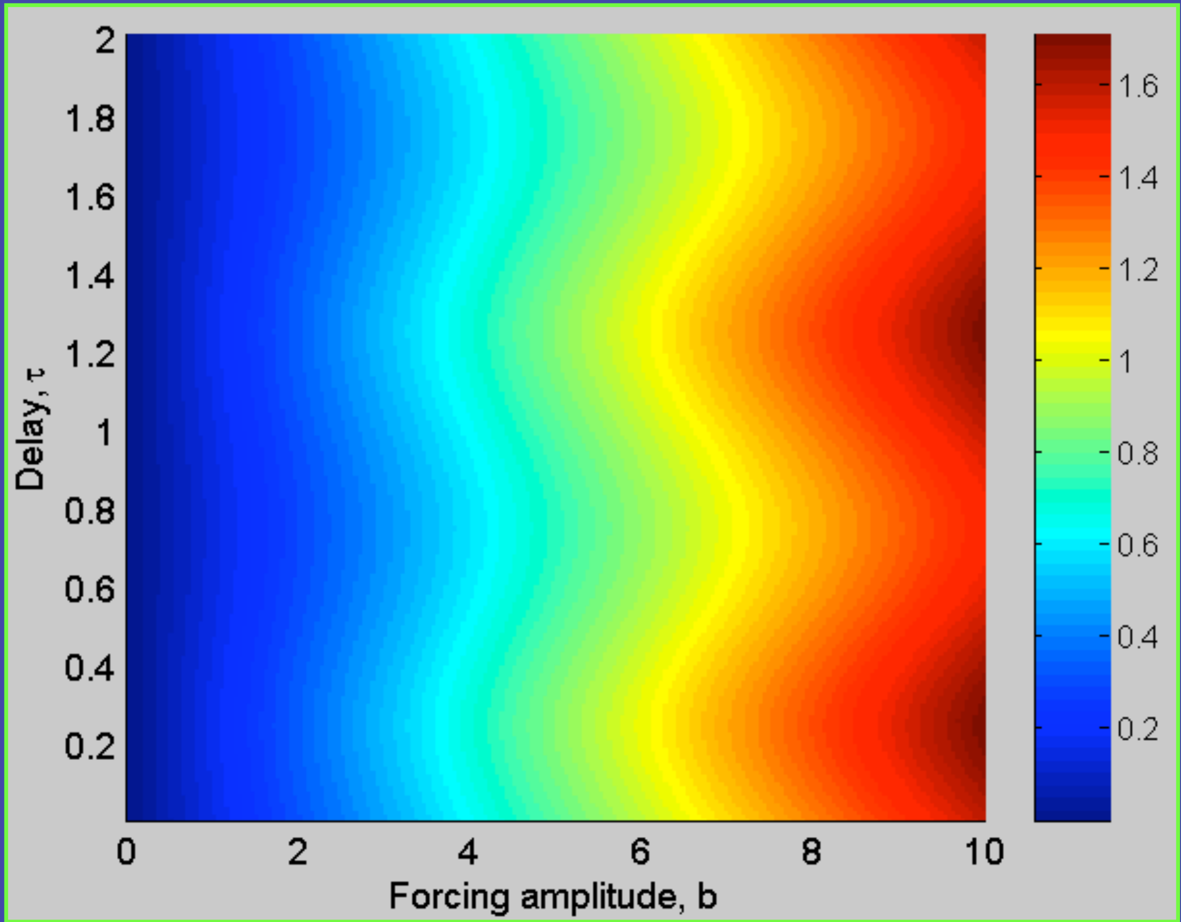
$b = 1,\ \kappa = 10,\ \tau = 0.45$

Interdecadal variability:
Spontaneous change of
(1) long-term annual mean, and
(2) Higher/lower positive and lower/higher negative extremes

N.B. Intrinsic, rather than forced!

Critical transitions (1)

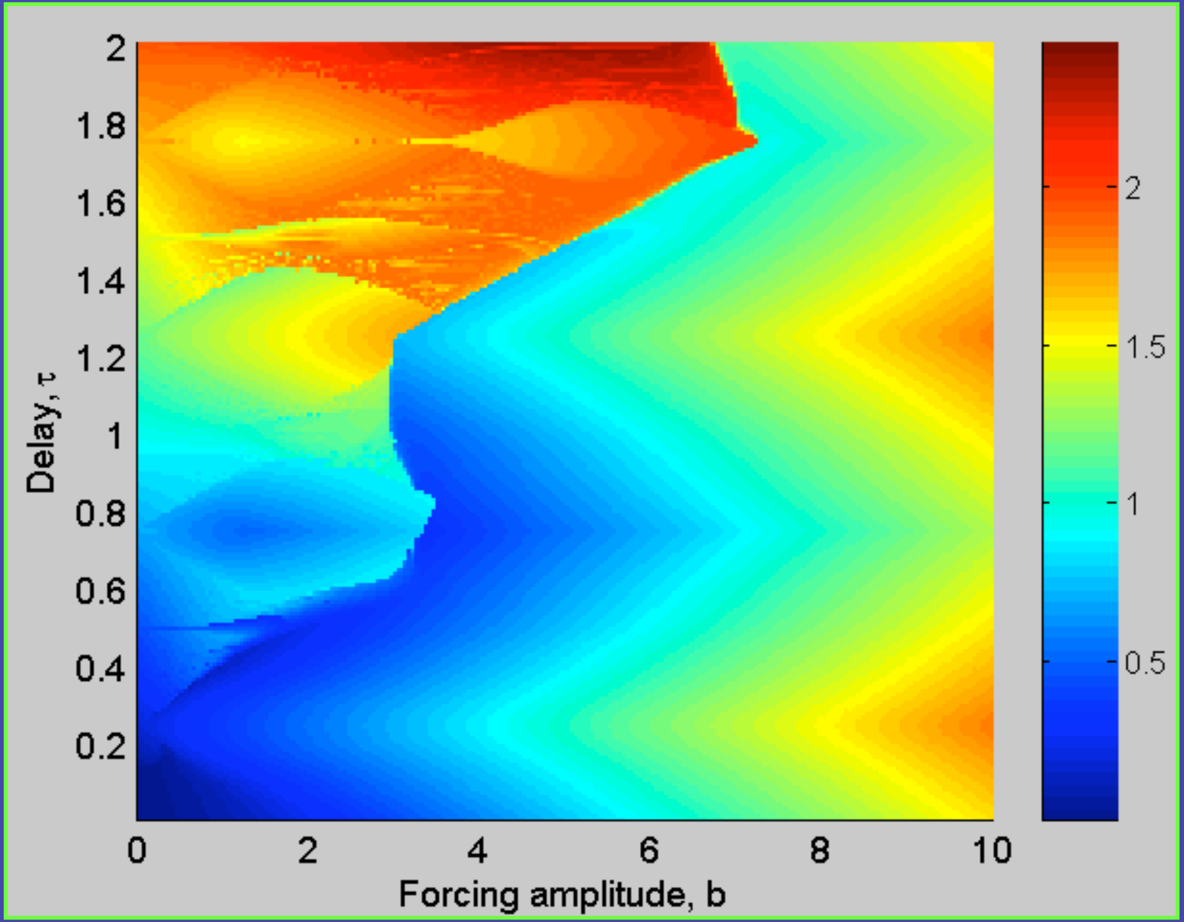
Trajectory maximum (after transient): $\kappa = 0.5$



- Smooth map
- Monotonic in *b*
- Periodic in τ

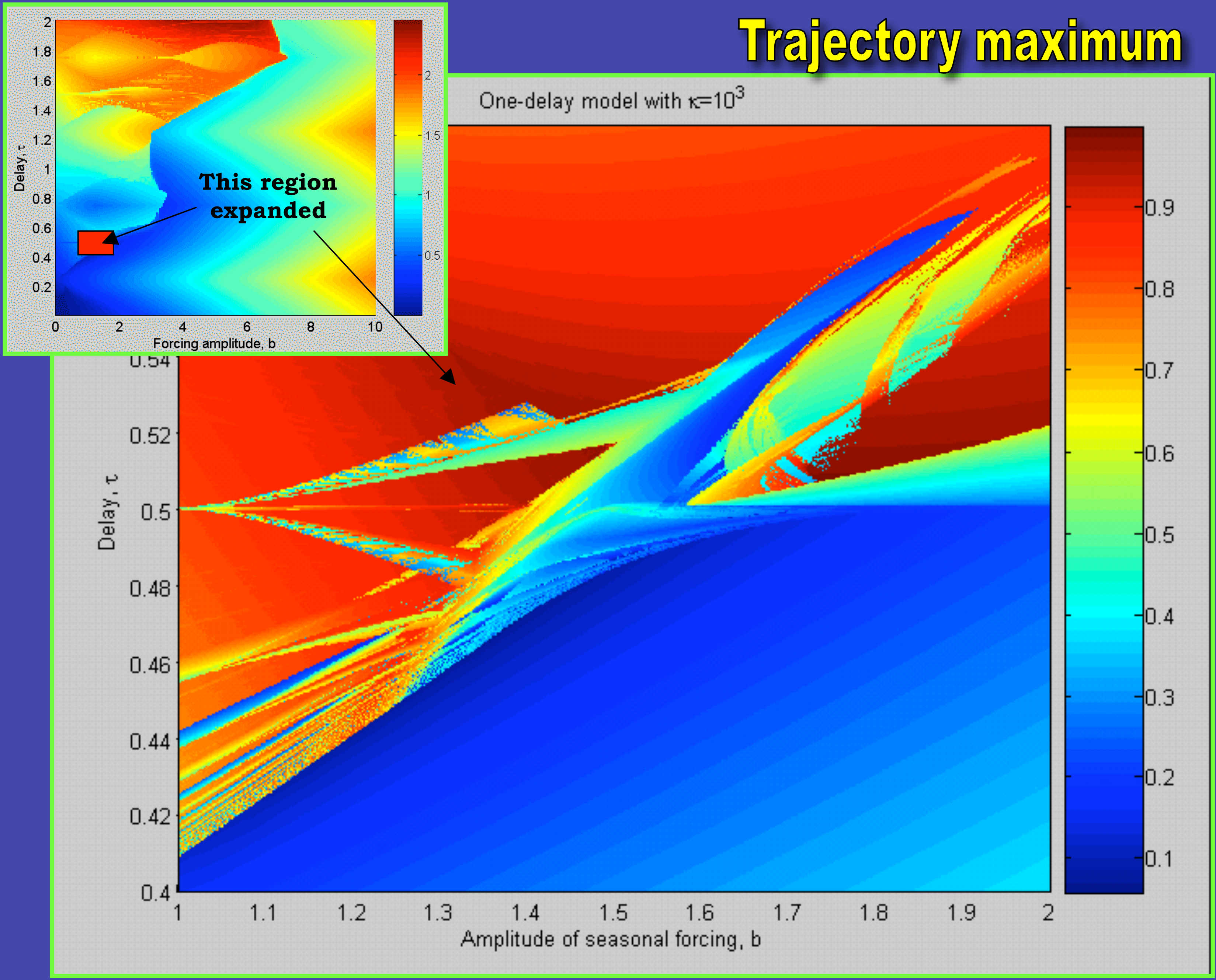
Critical transitions (4)

Trajectory maximum (after transient): $\kappa = 11$

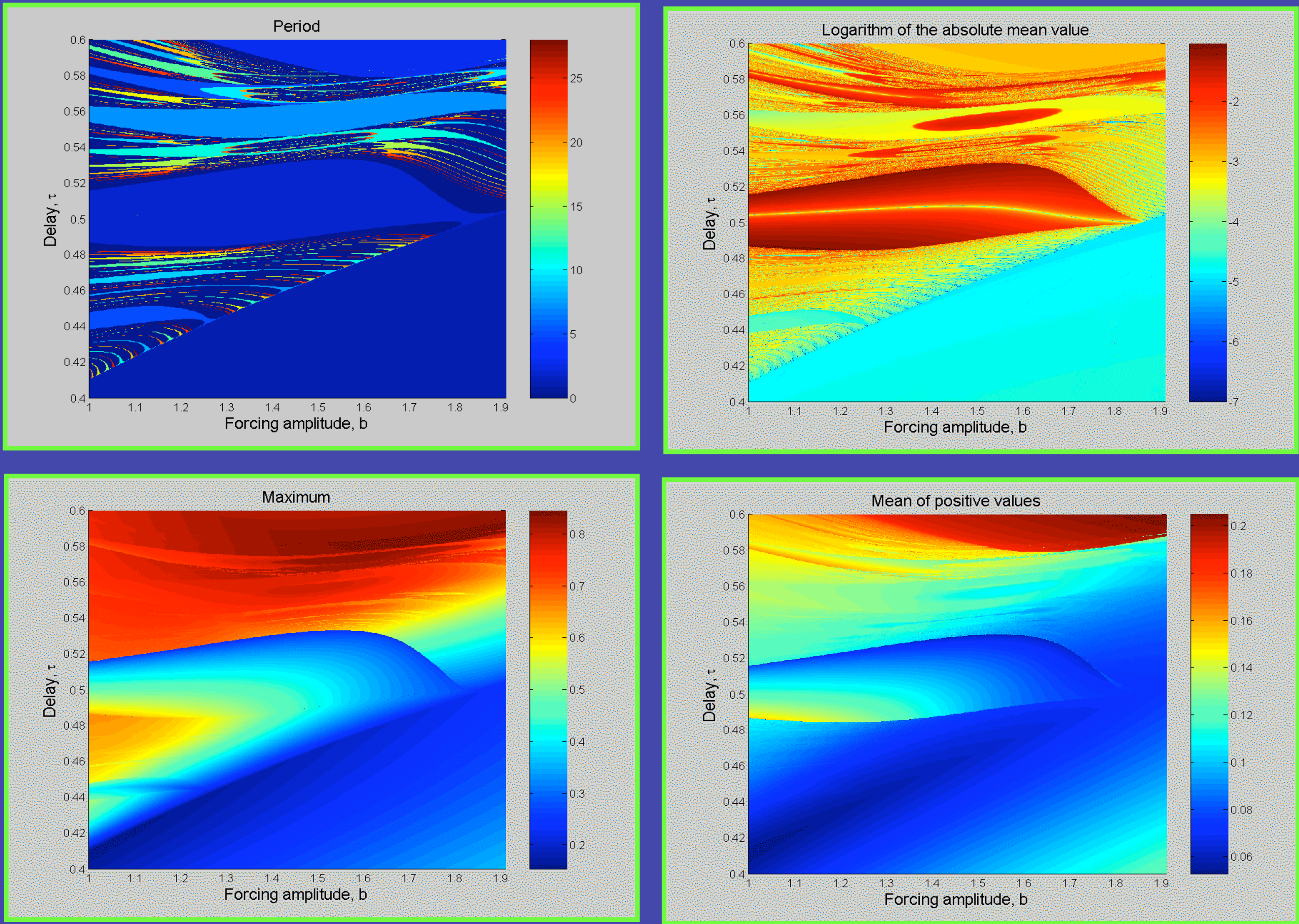


- The neutral curve moves to higher seasonal forcing *b* and lower delays τ.
- The neutral curve that separates rough from smooth behavior becomes itself crinkled (rough, fractal?).

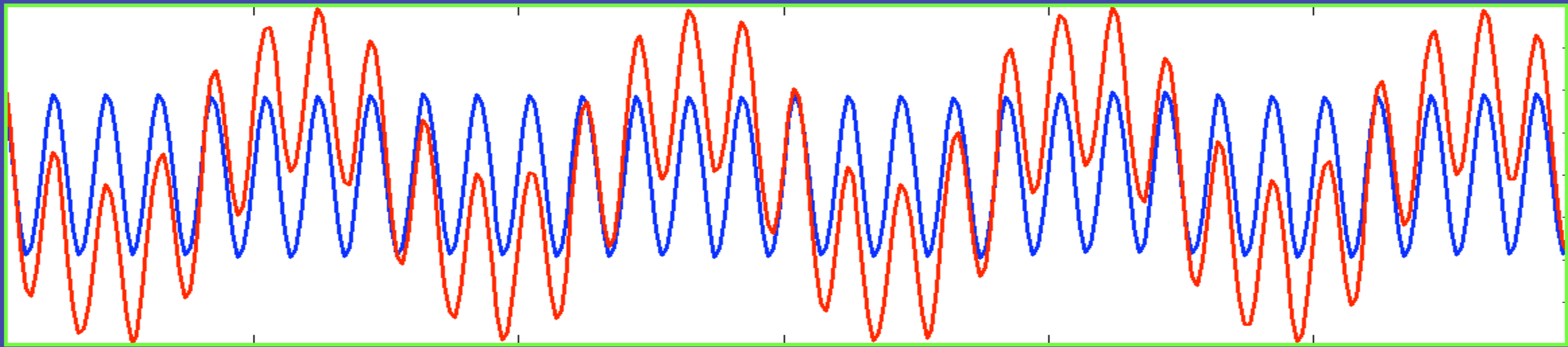
Trajectory maximum

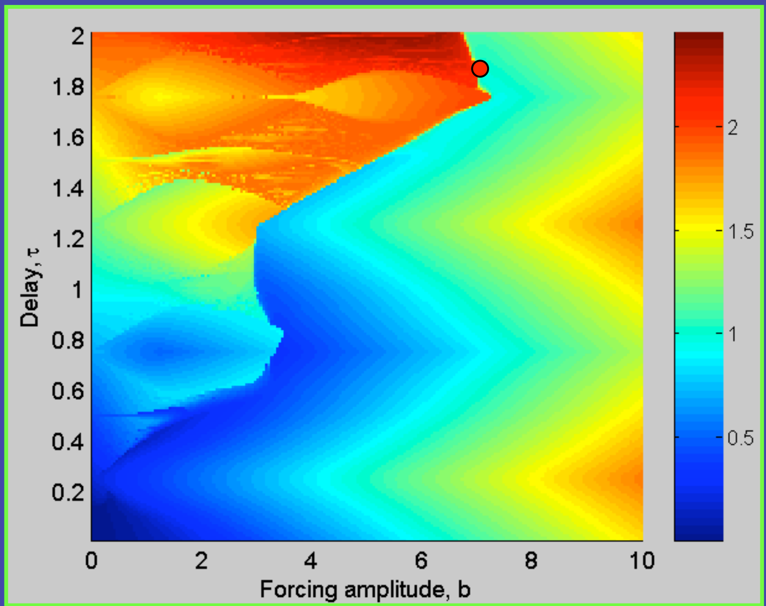


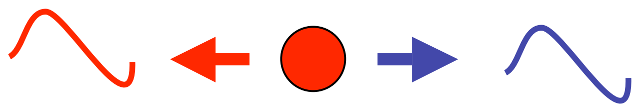
Intermediate forcing and delay



Example of instability







Can dynamical systems theory help, again?

The uncertainties might be ***intrinsic***, rather than mere "tuning problems"

If so, maybe ***stochastic structural stability*** could help!

Might fit in nicely with recent taste for "stochastic parameterizations"

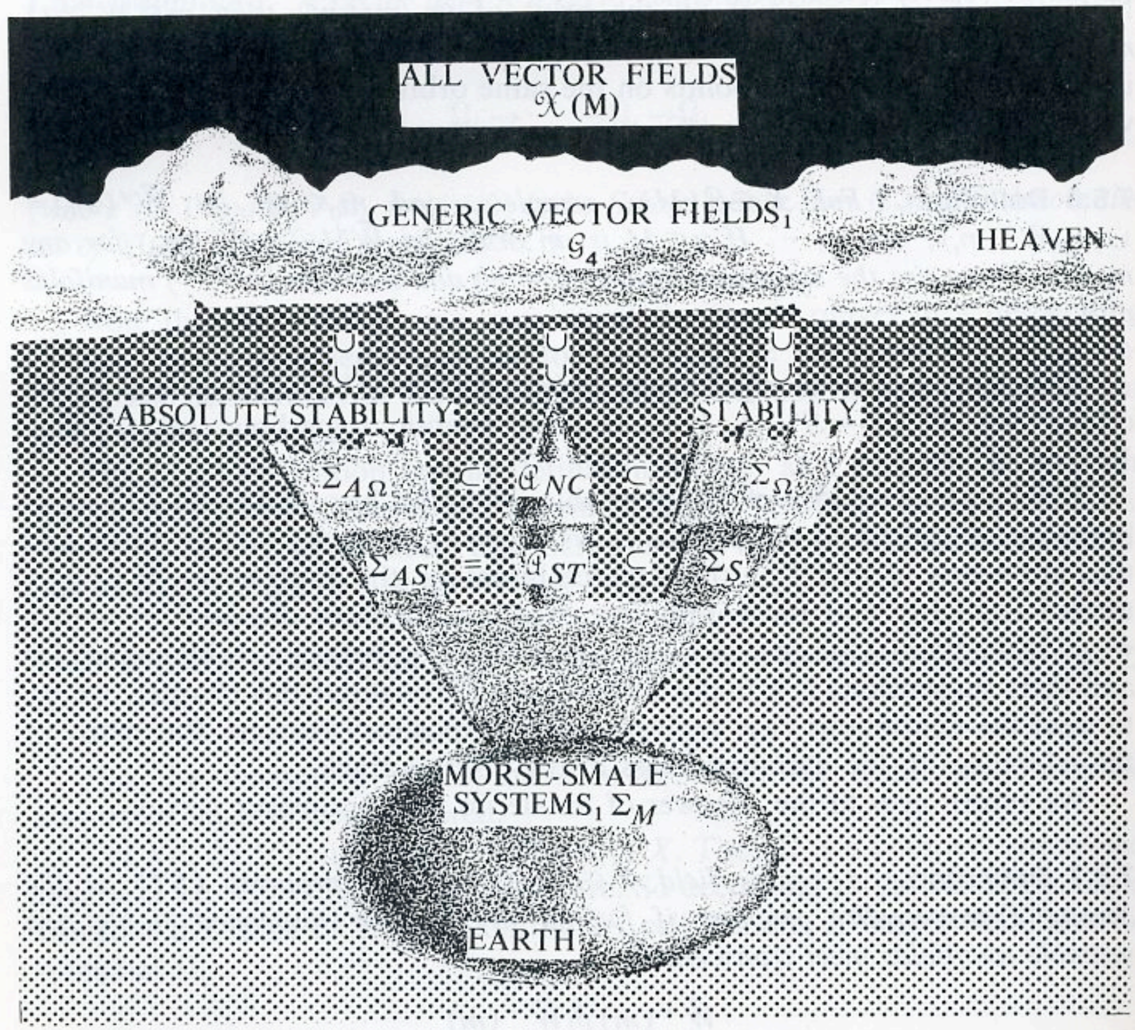


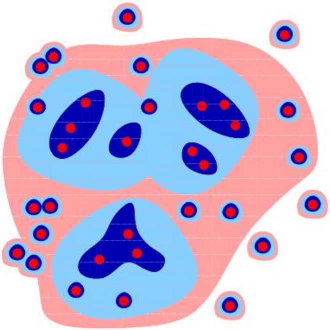
Figure 7.5-1. The three towers of differentiable dynamics.

The DDS dream of structural stability (from *Abraham & Marsden*, 1978)

So what's it gonna be like, by 2100?

Table SPM.2. *Recent trends, assessment of human influence on the trend and projections for extreme weather events for which there is an observed late-20th century trend.* {Tables 3.7, 3.8, 9.4; Sections 3.8, 5.5, 9.7, 11.2–11.9}

Phenomenon[a] and direction of trend	Likelihood that trend occurred in late 20th century (typically post 1960)	Likelihood of a human contribution to observed trend[b]	Likelihood of future trends based on projections for 21st century using SRES scenarios
Warmer and fewer cold days and nights over most land areas	*Very likely*[c]	*Likely*[d]	*Virtually certain*[d]
Warmer and more frequent hot days and nights over most land areas	*Very likely*[e]	*Likely (nights)*[d]	*Virtually certain*[d]
Warm spells/heat waves. Frequency increases over most land areas	*Likely*	*More likely than not*[f]	*Very likely*
Heavy precipitation events. Frequency (or proportion of total rainfall from heavy falls) increases over most areas	*Likely*	*More likely than not*[f]	*Very likely*
Area affected by droughts increases	*Likely* in many regions since 1970s	*More likely than not*	*Likely*
Intense tropical cyclone activity increases	*Likely* in some regions since 1970	*More likely than not*[f]	*Likely*
Increased incidence of extreme high sea level (excludes tsunamis)[g]	*Likely*	*More likely than not*[f,h]	*Likely*[i]



As the noise tends to zero or the parametrizations are switched off, structural instability reappears, as a “granularity" of model space. For nonzero variance, the random attractor $\mathcal{A}(\omega)$ associated with several GCMs might fall into larger and larger classes,

as the noise level increases.

Framework

This theory provides concepts and tools to deal rigorously with geometric aspects of stochastic dynamical systems, including SDEs. It provides the counterpart of the geometric theory of ODEs.

- RDS theory extends the notion of flows, via the concept of cocycle that models the stochastic trajectories (paths).
- RDS theory allows one to compare qualitative behavior between two systems, through a rigorous definition of a random change of variables in phase space.
- The concept of attraction is understood in a pullback sense, which leads to the concept of random attractor, taking into account the random character of the forcing.

The model

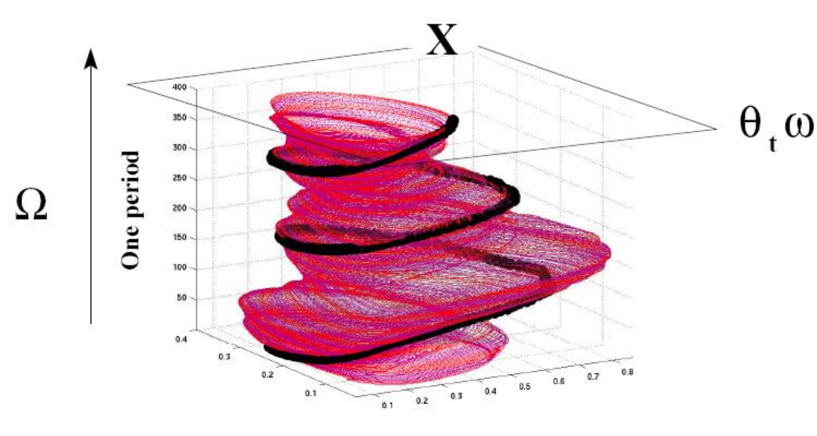
- We consider the following perturbed Holling's model (*Mem. Entomol. Soc. Canada*, 1965):

$$\begin{aligned} \dot{x} &= (r + \sigma\dot{\xi}_t)x(\alpha + x)(1 - x) - Cxy, \\ \dot{y} &= -\alpha dy + (C - d)xy, \end{aligned}$$

where

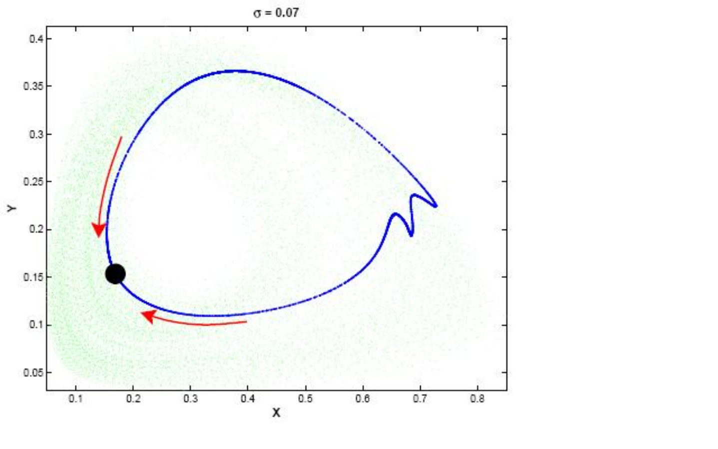
- x is the prey and y the predator,
- r and d: growth rate and death rate,
- $C = C_0 + \frac{(C_0 - d)}{4}\sin(\nu t)$ is a periodic coupling parameter that mimics a seasonally dependent hunting of x by y,
- $\sigma\dot{\xi}_t$: white noise of amplitude σ,
- α: bifurcation parameter.

A continuum of global random attractors (red tube) over one period.
The trajectories are all attracted by a random point shown in black.



- Note 3 : 1 subharmonic resonance!

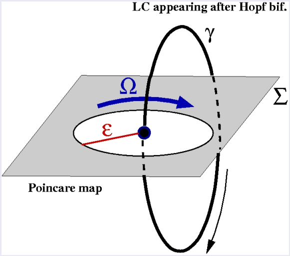
A section of the tube: the global attractor (**blue**), the attracting random point on it (**black**), and a single noisy trajectory (green dots)



Arnol'd family of diffeomorphisms

- We want to perform a *classification* in terms of stochastic equivalence.
- Our first theoretical laboratory is the Arnol'd family of circle maps:

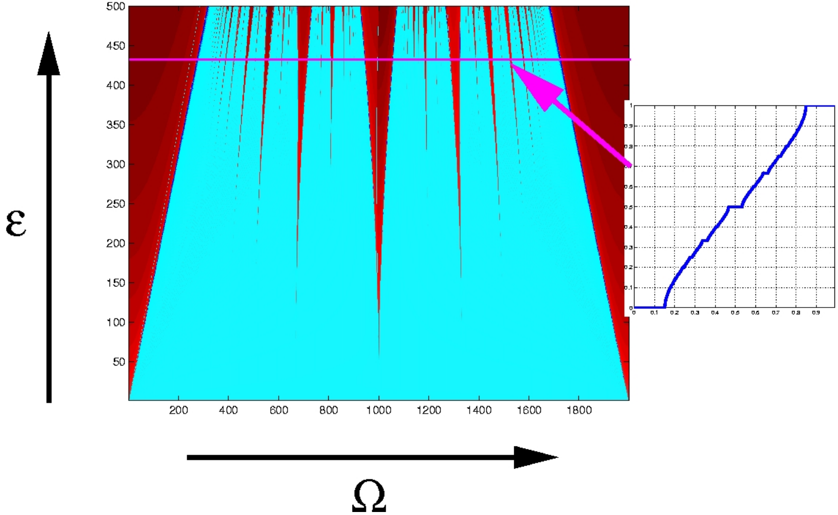
$$x_{n+1} = F_{\Omega,\varepsilon}(x_n) := x_n + \Omega - \varepsilon \sin(2\pi x_n) \mod 1$$



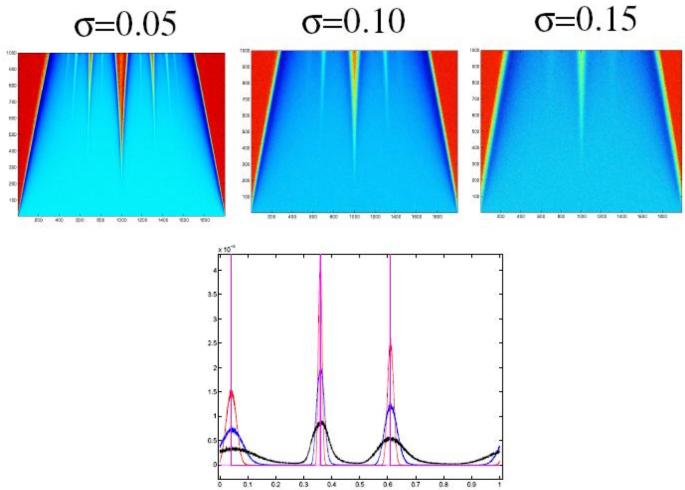
Why this family?

- Frequency-locking phenomena & the Devil's staircase
- **Topological classification** of the Arnol'd family $\{F_{\Omega,\varepsilon}\}$:
  - **countable** regions of structural stability;
  - **uncountable** structurally unstable systems, with **non-zero Lebesgue measure!**
- **Two types of attractors:**
  - periodic orbits on the circle;
  - the whole circle.

Arnol'd tongues and the Devil's staircase



Noise effects on topological classification



Effect of the noise on the PDF of the Arnol'd tongue 1/3

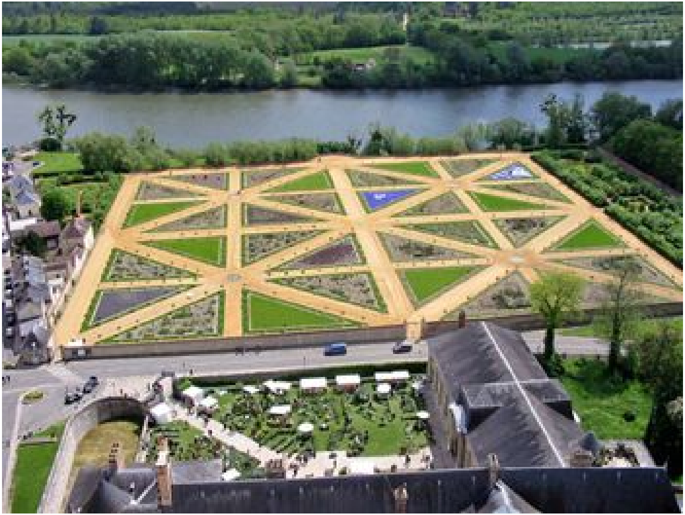
The deterministic model

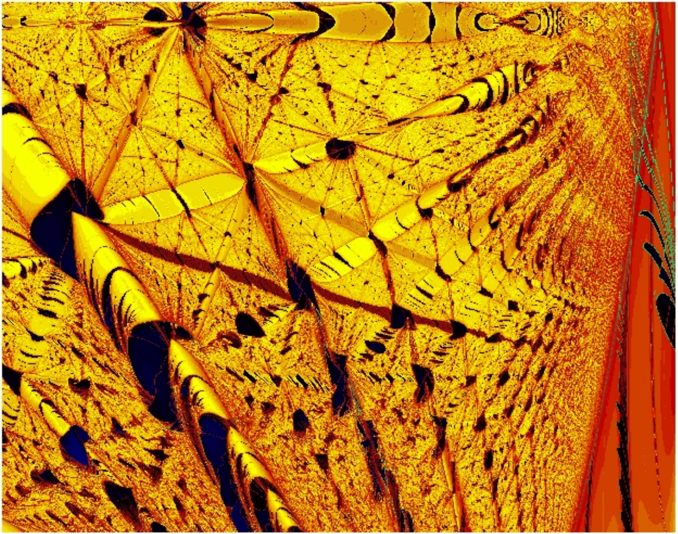
- Dynamics on a 2-D torus:

$$\begin{aligned} x_{n+1} &= x_n + \Omega_1 - \varepsilon \sin(2\pi y_n), \quad \text{mod } 1 \\ y_{n+1} &= y_n + \Omega_2 - \varepsilon \sin(2\pi x_n) \quad \text{mod } 1 \end{aligned}$$

- Web of resonances & chaos:
  - partial resonance — Ω_1 and Ω_2 are rational and there is a rational relation $m_1\Omega_1 + m_2\Omega_2 = k$; m_1, m_2, and k are integers
  - full resonance and chaos, with possibly multiple attractors
- A more realistic paradigm for dynamics observed in the geosciences and elsewhere.
- What is the effect of noise in such a context?

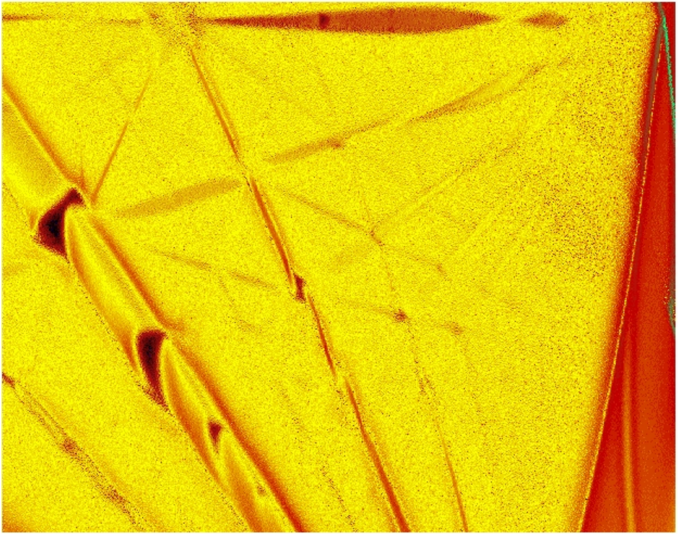
A French garden near the castle of La Roche-Guyon





- **coupling parameter** $\varepsilon = 0.15$

The Devil's quarry - noise effects



Computational issues

How to improve predictive understanding – past & present

- Increase resolution ~ O(N^3)
- Add or improve "physics" – highly variable cost
- Parameter estimation via trial-and-error ~ O(N^2–N^3)
- Use ensembles to reduce uncertainties ~ O(kN)

All of this helps, but doesn't really get us there – see AR4!

Dynamical systems issues – present & future

- Numerical bifurcation studies ~ O(N^2–N^4)
- Parameter estimation via data assimilation ~ O(N^2–N^4)
- Stochastic parametrizations ~ O(N^2–N^3)
- Random attractors ~ ??

Looks promising – trade-off between resolution N and insight?

With many thanks to DoE's CCPP and SciDAC programs …

Some conclusions &/or questions

What do we know?

- It's getting warmer.
- We do contribute to it.
- So, we should act as best we know and can!

What do we know less well?

- How does the climate system really work?
- How does natural variability interact with
  anthropogenic forcing?

What to do?

- Better understand the system and its forcings.
- Better understand the effects on economy and society,
  and vice-versa.
- Explore the models', and system's, stochastic structural stability.

Some general references

Andronov, A.A., and L.S. Pontryagin, 1937: Systèmes grossiers. *Dokl. Akad. Nauk. SSSR*, **14**(5), 247–250.

Arnold, L., 1998: *Random Dynamical Systems,* Springer Monographs in Math., Springer, 625 pp.

Charney, J., *et al.*, 1979: *Carbon Dioxide and Climate: A Scientific Assesment.* National Academies Press, Washington, D.C.

Dijkstra, H.A., 2005: *Nonlinear Physical Oceanography: A Dynamical Systems Approach to the Large-Scale Ocean Circulation and El Niño* (2nd edn.), Springer, 532 pp.

Ghil, M., R. Benzi, and G. Parisi (Eds.), 1985: *Turbulence and Predictability in Geophysical Fluid Dynamics and Climate Dynamics*, North-Holland,, 449 pp.

Ghil, M., and S. Childress, 1987: *Topics in Geophysical Fluid Dynamics: Atmospheric Dynamics, Dynamo Theory and Climate Dynamics*, Springer, 485 pp.

Ghil, M., 2001: Hilbert problems for the geosciences in the 21st century, *Nonlin.Proc. Geophys.,* **8**, 211–222.

Ghil, M., and E. Simonnet, 2008: *Nonlinear Climate Theory*, Cambridge Univ. Press, Cambridge, UK/London/New York, in preparation (approx. 450 pp.).

Houghton, J. T., G. J. Jenkins, and J. J. Ephraums (Eds.), 1991: *Climate Change, The IPCC Scientific Assessment*, Cambridge Univ. Press, Cambridge, MA, 365 pp.

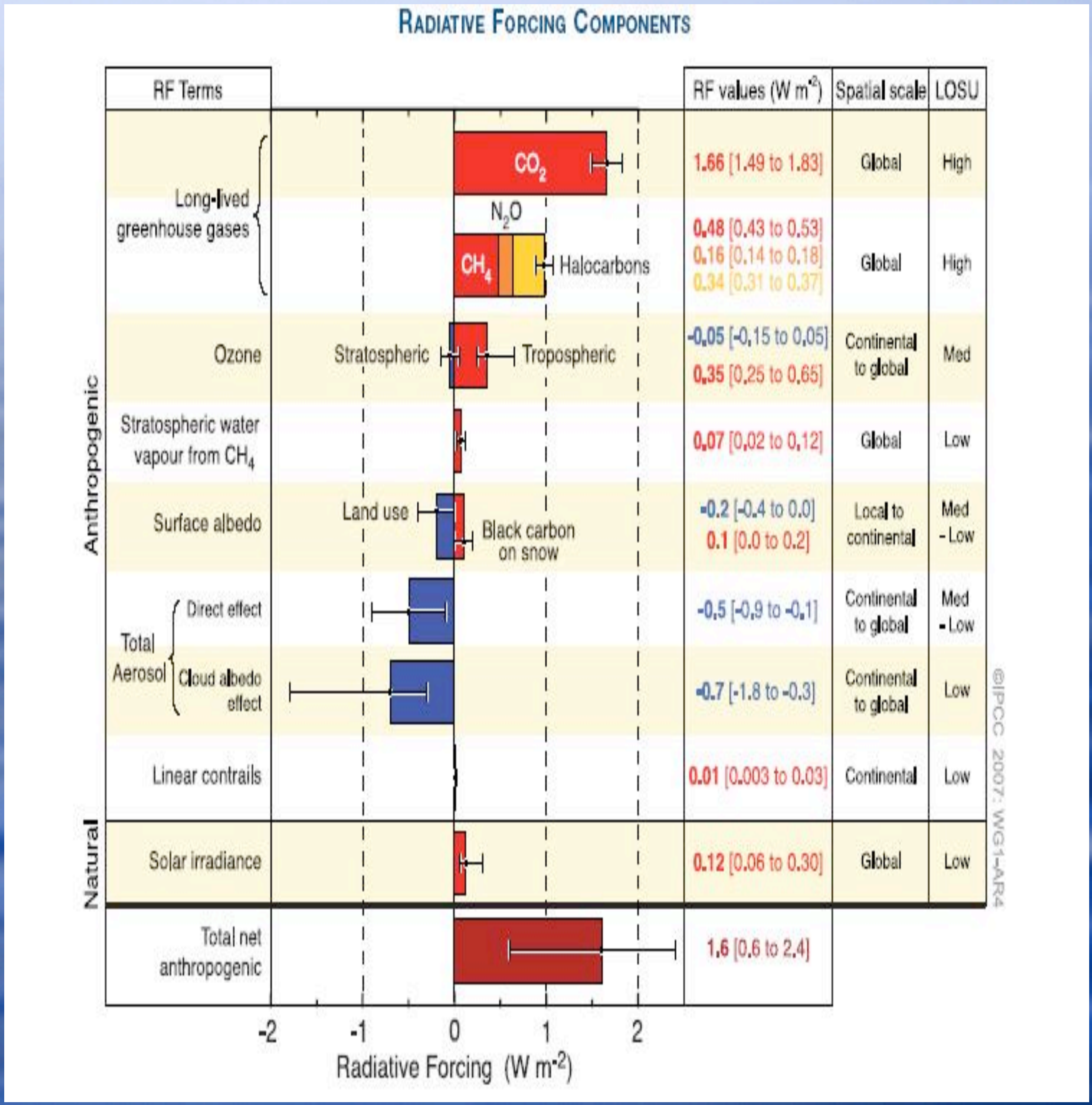
Solomon, S., *et al.* (Eds.). *Climate Change 2007: The Physical Science Basis. Contribution of Working Group I to the Fourth Assessment Report of the IPCC*, Cambridge Univ. Press, Cambridge, UK and New York, NY, USA, 2007.

Reserve slides

GHGs rise

It's gotta do with us, at least a bit, ain't it?

But just how much?



IPCC (2007)

Unfortunately, things aren’t all that easy!

What to do?

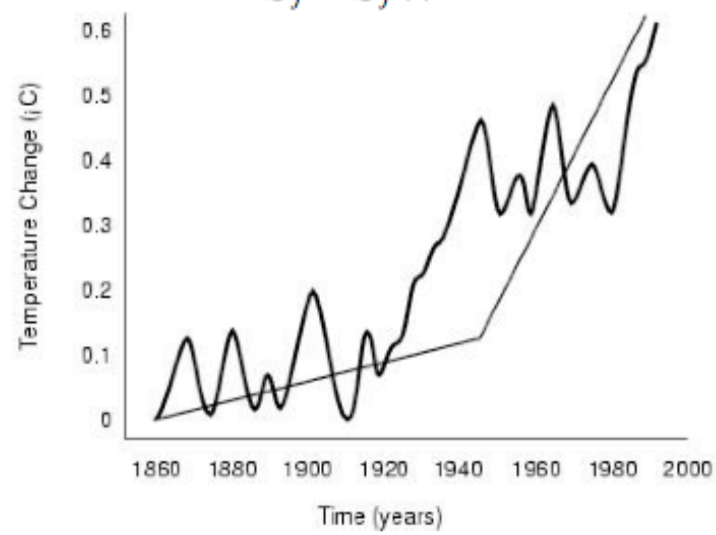
Try to achieve better interpretation of, and agreement between, models …

Ghil, M., 2002: Natural climate variability, in *Encyclopedia of Global Environmental Change*, T. Munn (Ed.), Vol. 1, Wiley

Natural variability introduces additional complexity into the anthropogenic climate change problem

The most common interpretation of observations and GCM simulations of climate change is still in terms of a scalar, linear Ordinary Differential Equation (ODE)

$$c\frac{dT}{dt} = -kT + Q \qquad \begin{aligned} k &= \sum k_i - \text{feedbacks (+ve and -ve)} \\ Q &= \sum Q_j - \text{sources \& sinks} \\ Q_j &= Q_j(t) \end{aligned}$$



Linear response to CO_2 vs. observed change in T

Hence, we need to consider instead a system of nonlinear Partial Differential Equations (PDEs), with parameters and multiplicative, as well as additive forcing (deterministic + stochastic)

$$\frac{d\boldsymbol{X}}{dt} = \boldsymbol{N}(\boldsymbol{X}, t, \mu, \beta)$$

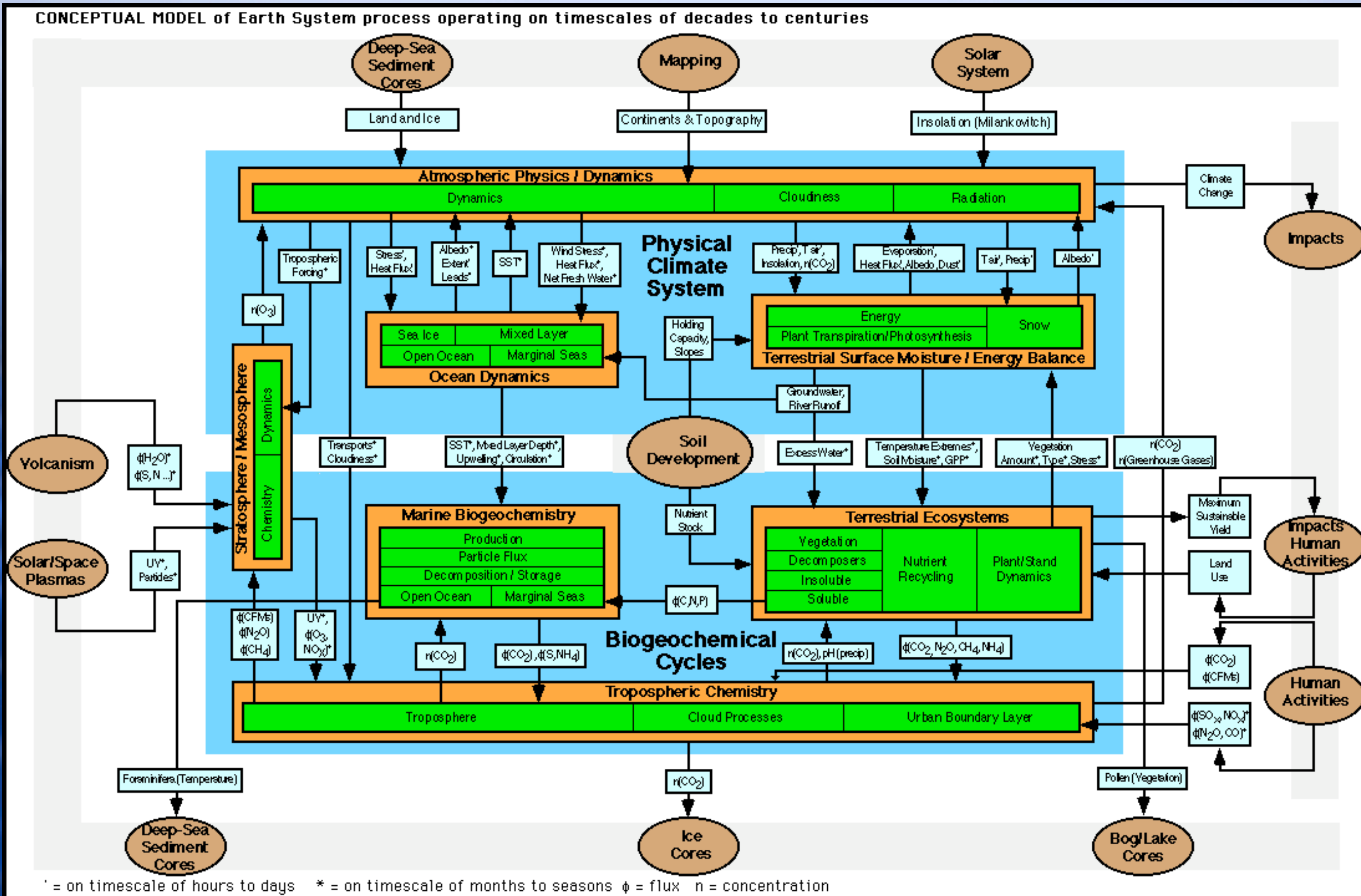
Climatic uncertainties & moral dilemmas

♥ … *keep today's climate for tomorrow?*

♥ *Feed the world today or…*

Davos, Feb. 2008, photos by *TIME Magazine*, 11 Feb. '08;
see also Hillerbrand & Ghil, *Physica D*, 2008, in press.

F. Bretherton's "horrendogram" of Earth System Science

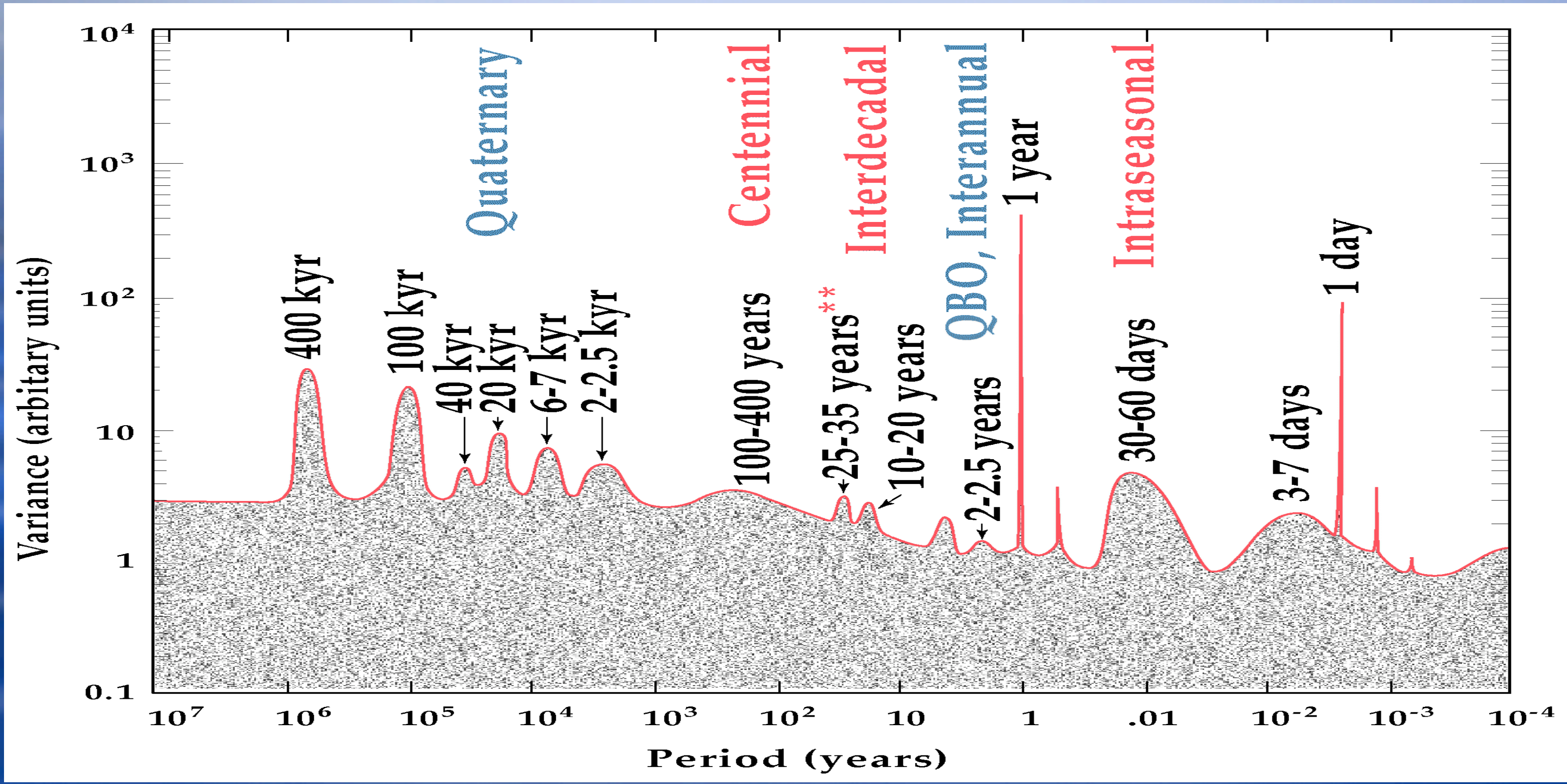


Earth System Science Overview, *NASA Advisory Council*, 1986

Composite spectrum of climate variability

Standard treatement of frequency bands:

1. ***High frequencies – white (or "colored") noise***
2. ***Low frequencies – slow ("adiabatic") evolution of parameters***



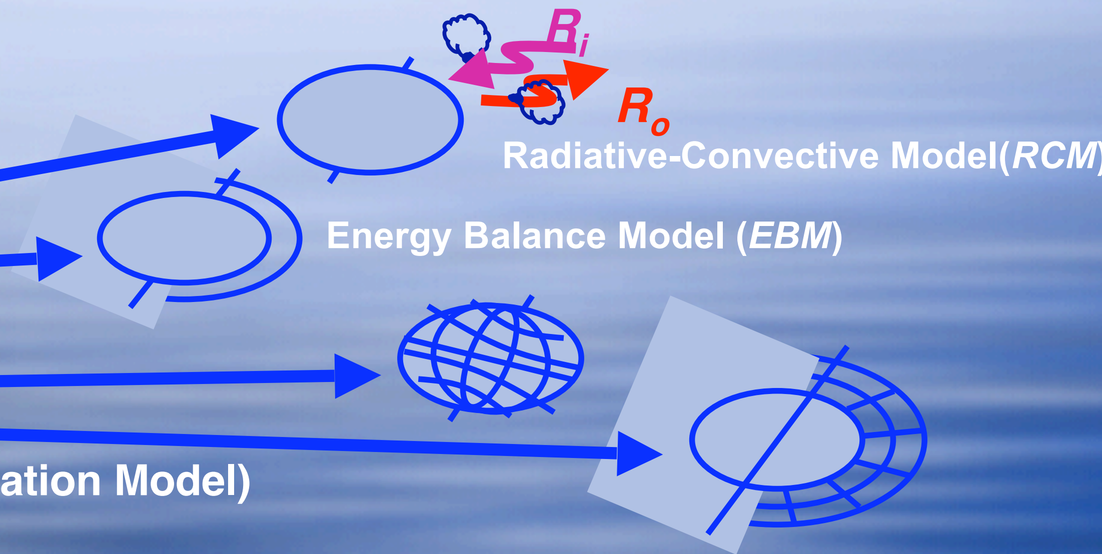
From **Ghil (2001, EGEC),** after **Mitchell* (1976)**

* "**No known source of deterministic internal variability**"

Climate models (atmospheric & coupled) : A classification

- *Temporal*
  - stationary, (quasi-)equilibrium
  - transient, climate variability

- *Space*
  - 0-D (dimension 0)
  - 1-D
    - vertical
    - latitudinal
  - 2-D
    - horizontal
    - meridional plane
  - 3-D, GCMs (General Circulation Model)
    - horizontal
    - meridional plane
  - Simple and intermediate 2-D & 3-D models

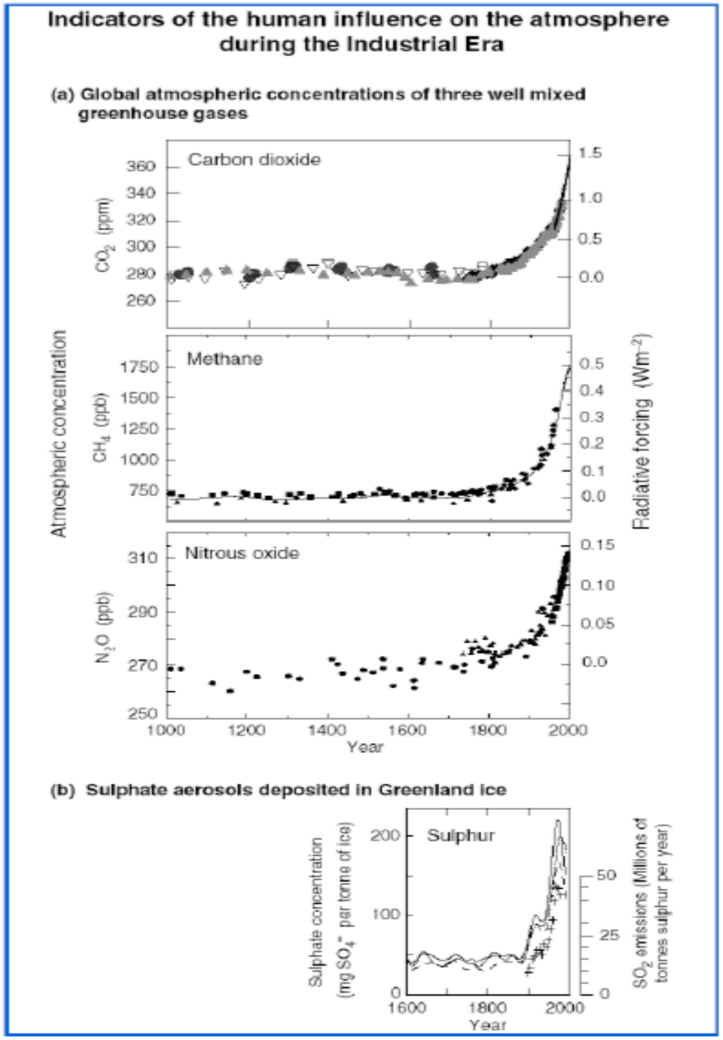


- *Coupling*
  - Partial
    - unidirectional
    - asynchronous, hybrid
  - Full

Hierarchy: from the simplest to the most elaborate,
iterative comparison with the observational data

GHGs rise

It's gotta do with us, at least a bit, ain't it?



IPCC (2001)

Delay models of ENSO variability

Battisti & Hirst (1989)

$$dT/dt = -\alpha T(t-\tau) + T, \ \ \alpha > 0, \tau > 0$$

Suarez & Schopf (1988), Battisti & Hirst (1989)

$$dT/dt = -\alpha T(t-\tau) + T - T^3$$

Tziperman *et al.* (1994)

$$dT/dt = -\alpha \tanh\left[\kappa T(t-\tau_1)\right] + \beta \tanh\left[\kappa T(t-\tau_2)\right] + \gamma \cos(2\pi t)$$

Atmosphere–ocean coupling (Munnich *et al.*, 1991)

Seasonal forcing

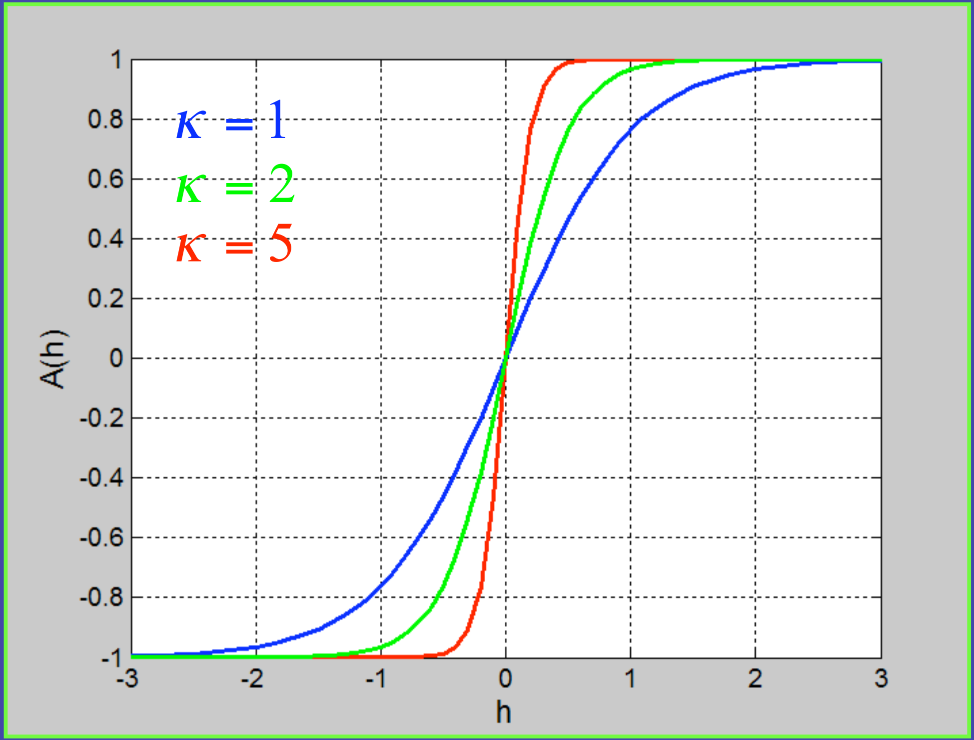
Model parameters:

$$\frac{d}{dt}h(t) = -A\left[h(t-\tau)\right] + b\cos(2\pi\omega t)$$

$$A(h) = \tanh(\kappa h)$$

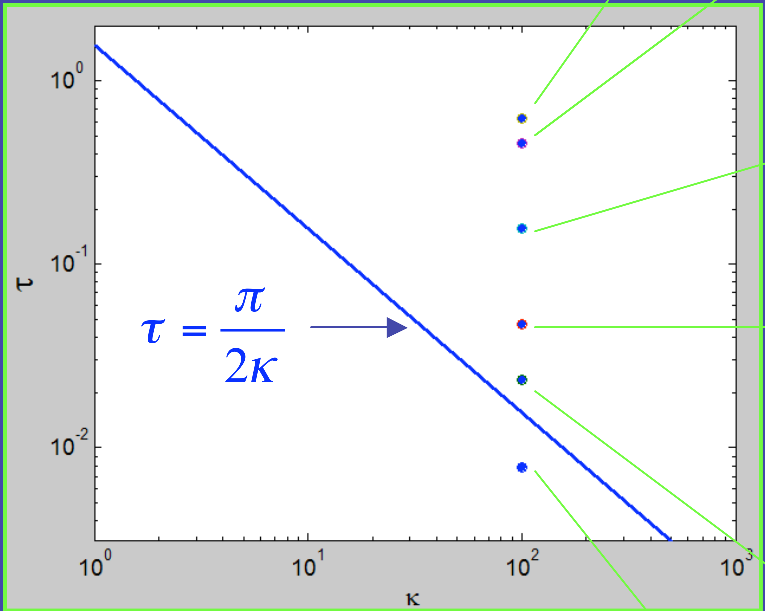
Wind-forced ocean waves (Kelvin, Rossby)

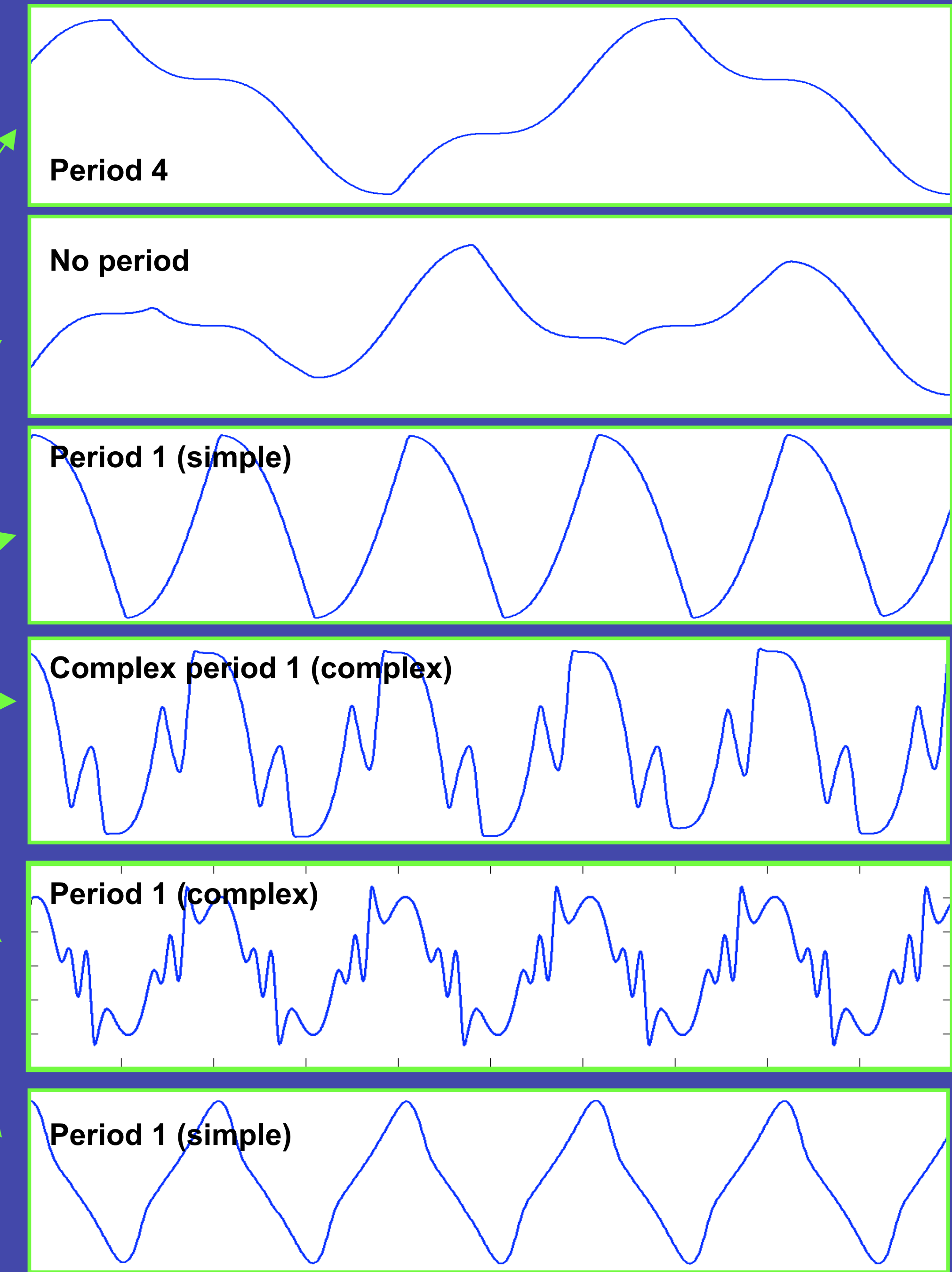
Strength of the atmosphere-ocean coupling



Examples

$$b = 1,\ \kappa = 100$$





Existence, uniqueness, continuous dependence

$$\frac{dh(t)}{dt} = -\tanh\left[\kappa h(t-\tau)\right] + b\cos(2\pi t),\ t \ge 0 \qquad (1)$$

$$h(t) = \varphi(t),\ t \in [-\tau, 0) \qquad (2)$$

Theorem

The IVP (1-2) has a unique solution on $[0,\infty)$ for any set $(\kappa, b, \tau, \varphi)$. This solution depends continuously on initial data $\varphi(t)$, delay τ, and the rhs of (1) (in an appropriate norm).

Corollary

A **discontinuity** in solution profile indicates existence of an **unstable solution** that separates attractor basins of two stable ones.

Concluding remarks

1. A ***simple differential-delay equation (DDE)*** with a ***single delay*** reproduces the ***realistic scenarios*** documented in other ENSO models, such as nonlinear PDEs and GCMs, as well as in observations.

2. The model illustrates well ***the role of the distinct parameters***: seasonal forcing b, ocean-atmosphere coupling κ, and oceanic wave delay τ.

3. ***Spontaneous transitions*** in mean temperature, as well as ***in extreme annual values*** occur, for purely periodic, seasonal forcing.

4. A sharp ***neutral curve*** in the (b–τ) plane ***separates smooth behavior*** of the period map from ***"rough" behavior.***

5. The model's dynamics is governed by multiple ***(un)stable solutions***; location of stable solutions in parameter space is intermittent.

6. The local ***extrema are locked*** to a particular season in the annual cycle.

7. We expect such ***behavior in much more detailed and realistic models***, where it is harder to describe its causes as completely.

References for ENSO-FDE model

o Ghil, M., and A. W. Robertson, 2000: Solving problems with GCMs: General circulation models and their role in the climate modeling hierarchy. *General Circulation Model Development: Past, Present and Future,* D. Randall (Ed.), Academic Press, San Diego, pp. 285–325.

o Hale, J. K., 1977: *Theory of Functional Differential Equations*, Springer-Verlag, New York, 365 pp.

o Jin, F.-f., J. D. Neelin and M. Ghil, 1994: El Niño on the Devil's Staircase: Annual subharmonic steps to chaos, *Science*, **264**, 70–72.

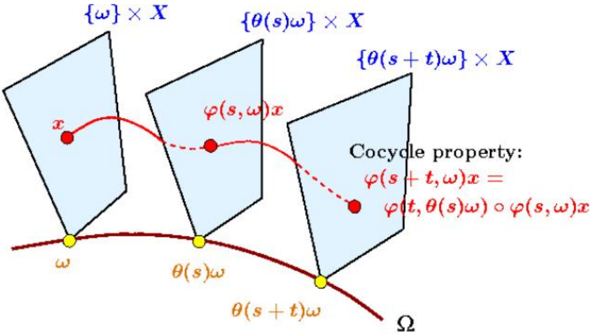
o Saunders, A., and M. Ghil, 2001: A Boolean delay equation model of ENSO variability, *Physica D*, **160**, 54–78.

o Tziperman, E., L. Stone, M. Cane and H. Jarosh, 1994: El Niño chaos: Overlapping of resonances between the seasonal cycle and the Pacific ocean-atmosphere oscillator. *Science*, **264**, 72–74.

o Munnich, M., M. Cane, and S. Zebiak, 1991: A study of self-excited oscillations of the tropical ocean atmosphere system 2. Nonlinear cases , *J. Atmos. Sci.,* **48**, 1238–1248.

A few details (“light")

- Noise forcing is modeled by a stationary process; its coupling with the underlying deterministic dynamical system (DDS) is expressed mathematically by the cocycle property.
- Fiber-by-fiber view of the dynamics: each fiber represents the phase space, parameterized by distinct realizations of the noise.
- A path of the stochastic process thus corresponds to a selection of points in each fiber of the resulting bundle; see next figure.



- φ **is a random dynamical system (RDS)**
- **The cocycle property is analogous to the semi-group property for DDS**
- $\Theta(t)(x,\omega) = (\theta(t)\omega, \varphi(t,\omega)x)$ **is a flow on the bundle**

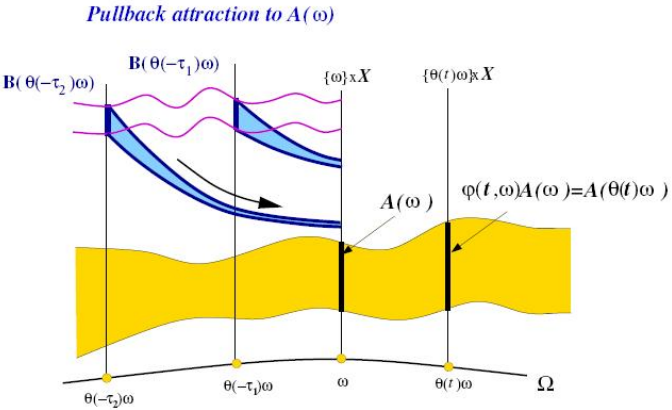
Key features

- Random attractors involve **pullback attraction** in a non-autonomous system.
- Pullback attraction does not involve running time backwards: we perform measurements at time t in an experiment started at time $s < t$ long ago; hence we assess the “attracting state" at time t.
- For random forcing, we get a random attractor; it represents the frozen statistics at time t, when a long-enough history is taken into account, and it evolves with time.
- These geometric objects are numerically computable.

A random attractor $\mathcal{A}(\omega)$ is both *invariant* and “pullback" *attracting*:

(a) **Invariant**: $\varphi(t,\omega)\mathcal{A}(\omega) = \mathcal{A}(\theta(t)\omega)$.

(b) **Attracting**: $\forall B \subset X$, $\lim_{t\to\infty} \mathrm{dist}(\varphi(t,\theta(-t)\omega)B, \mathcal{A}(\omega)) = 0$ almost surely



Concluding remarks

Some insights

- Reduction of the attractor dimension:
  $\lim_{\sigma \to 0} \dim \mathcal{A}_\sigma(\omega) < \dim \mathcal{A}_0$ as the noise intensity $\sigma \to 0$.
- Stochastic parametrization $\Rightarrow$ gain of structural stability for random attractors.
- These results hold for **relevant deterministic models** that are stochastically perturbed.
- RDS theory offers a meaningful framework for performing classification in stochastic modeling.

Future work

- Colored-noise and lag-correlation effects on stochastic classes.
- Noise effects on nonhyperbolic chaos (Lorenz system, Newhouse phenomena, Hénon map, etc.)

Comparison procedure of random dynamical systems

- We now want to use these tools in order to compare two cocycles φ_1 and φ_2 representing two RDSs or SDEs.
- To be qualitatively the same, these cocycles have to exhibit topologically the same random attractors.
- The time-dependent character of random attractors contrasts with the classical notion of probability density function (PDF), which is frozen in time.
- N.a.s.c. to be qualitatively the same is that there exist a random change of variables that transforms φ_1 into φ_2; that is, φ_1 and φ_2 should be stochastically equivalent.

A tool for classification: stochastic equivalence

- Stochastic equivalence: two cocycles $\varphi_1(t,\omega)$ and $\varphi_2(t,\omega)$ are conjugated iff there exists a **random homeomorphism** h of X and an invariant set $\tilde{\Omega}$ of full $\mathbb{P}$-measure (w.r.t. θ) such that $h(\omega)(0) = 0$ and:

$$\varphi_1(t,\omega) = h(\theta(t)\omega)^{-1} \circ \varphi_2(t,\omega) \circ h(\omega); \tag{1}$$

  h is also called a cohomology of φ_1 and φ_2.
  It is a random change of variables!
- Motivation: We would like to measure qualitatively, as well as quantitatively, the difference between climate models.

The "hockey stick" & beyond

The "hockey stick" of TAR (3rd Assesment Report) is a typically (over)simplified version of much more detailed and reliable knowledge.

National Research Council, 2006: *Surface Temperature Reconstructions For the Last 2000 Years*. National Academies Press, Washington, DC, 144 pp. http://www.nap.edu/openbook.php?record_id=11676&page=2

2 SURFACE TEMPERATURE RECONSTRUCTIONS FOR THE LAST 2,000 YEARS

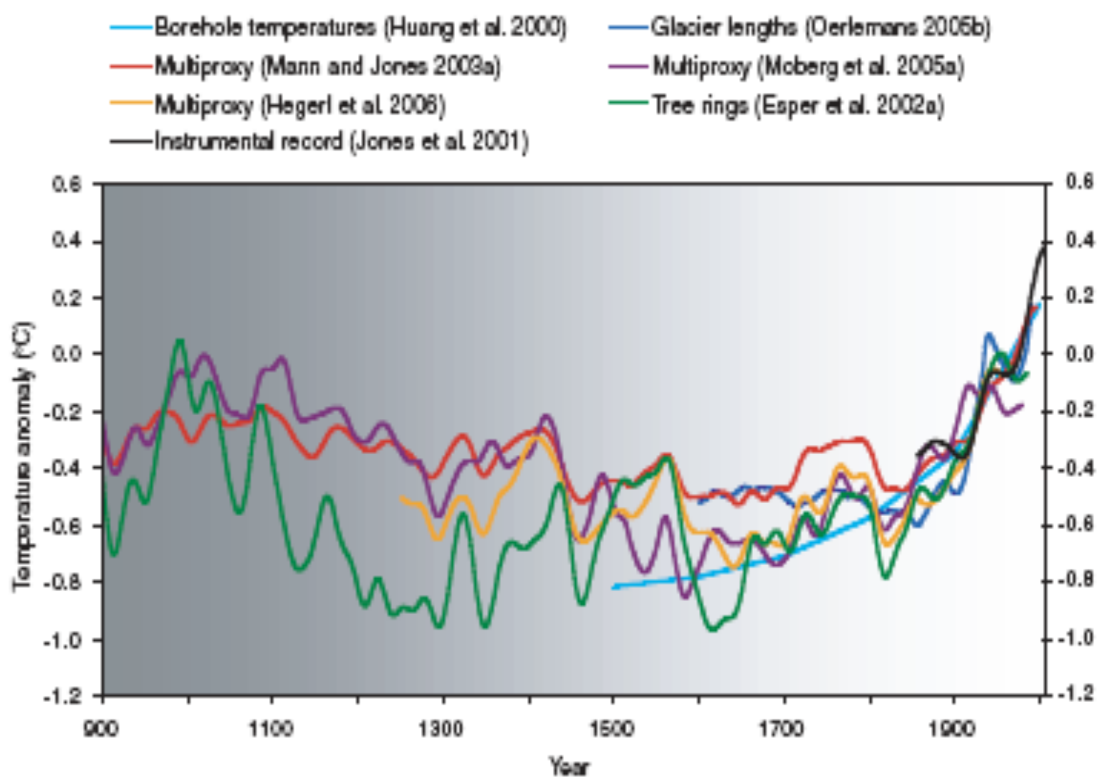
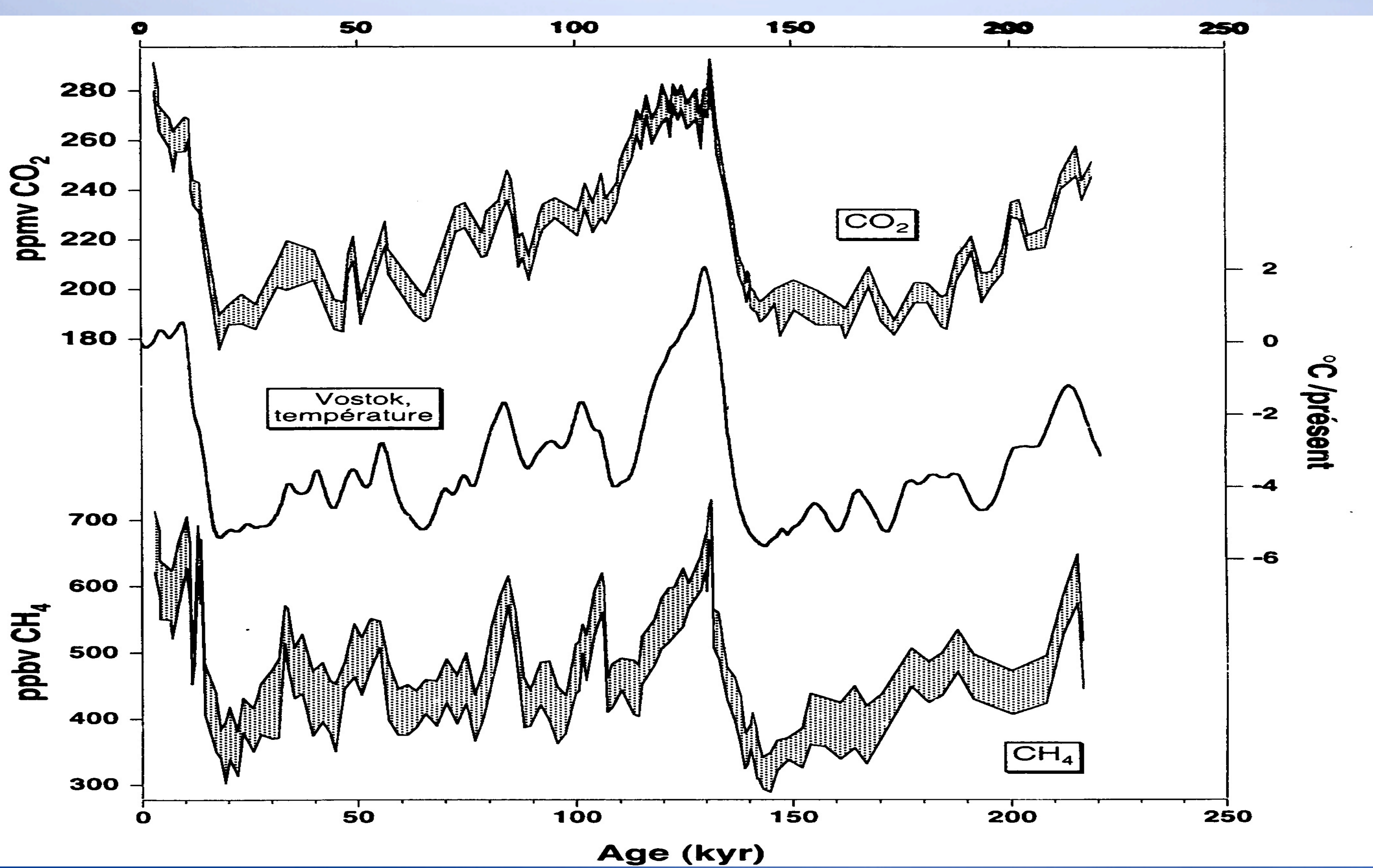
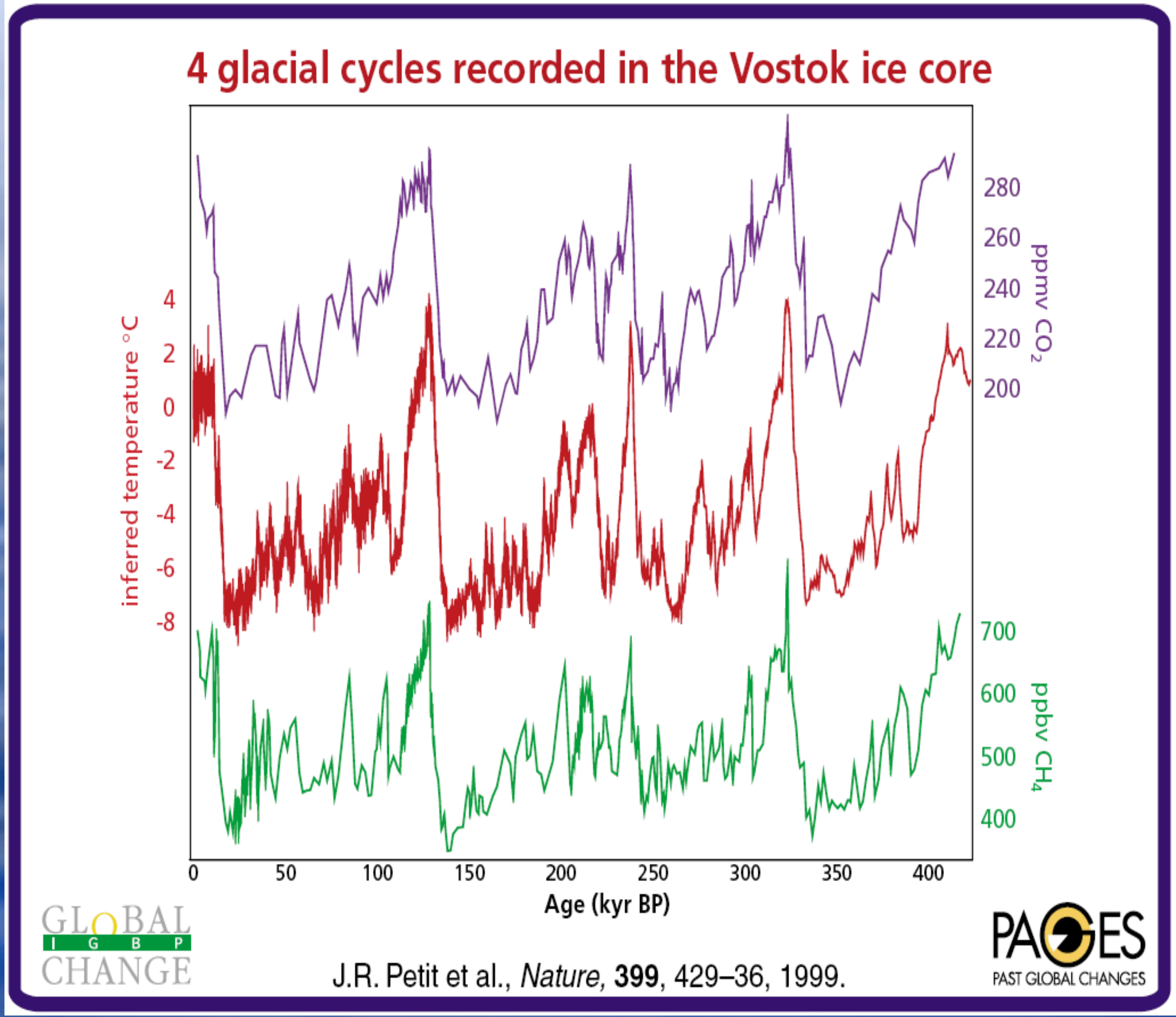


FIGURE S-1 Smoothed reconstructions of large-scale (Northern Hemisphere mean or global mean) surface temperature variations from six different research teams are shown along with the instrumental record of global mean surface temperature. Each curve portrays a somewhat different history of temperature variations and is subject to a somewhat different set of uncertainties that generally increase going backward in time (as indicated by the gray shading). This set of reconstructions conveys a qualitatively consistent picture of temperature changes over the last 1,100 years and especially over the last 400. See Figure O-5 for details about each curve.



Isotopic (proxy) temperatures and GHGs at Vostok, over the last glacial cycle; courtesy of **P. Yiou**

T_s and GHGs over 400 kyr



The same lead–lag relations are apparent over these 4 glacial cycles …

Sun-Climate Relations

- It ain't new: v. ~1000 papers (in 1978!), as well as Marcus *et al.* (1998, *GRL*).
- "Corrélation n'est pas raison."
- Requires serious study of solar physics.

Climatology Supplement

Nature 276, 348 - 352 (23 November 1978); doi:10.1038/276348a0

Solar–terrestrial influences on weather and climate

GEORGE L. SISCOE

Department of Atmospheric Sciences, University of California, Los Angeles, California 90024

During the past century over 1,000 articles have been published claiming or refuting a correlation between some aspect of solar activity and some feature of terrestrial weather or climate. Nevertheless, the sense of progress that should attend such an outpouring of 'results' has been absent for most of this period. The problem all along has been to separate a suspected Sun–weather signal from the characteristically noisy background of both systems. The present decade may be witnessing the first evidence of progress in this field. Three independent investigations have revealed what seem to be well resolved Sun–weather signals, although it is still too early to have unreserved confidence in all cases. The three correlations are between terrestrial climate and Maunder Minimum-type solar activity variations, a regional drought cycle and the 22-yr solar magnetic cycle, and winter hemisphere atmospheric circulation and passages by the Earth of solar sector boundaries in the solar wind. The apparent emergence of clear Sun–weather signals stimulated numerous searches for underlying physical causal links.

Extreme Events:
Causes and Consequences
(E2-C2)

- EC-funded project bringing together researchers in mathematics, physics, environmental and socio-economic sciences.
- €1.5M over three years (March 2005–Feb. 2008).
- Coordinating institute: Ecole Normale Supérieure.
- 17 'partners' in 9 countries.
- 72 scientists + 17 postdocs/postgrads.
- PEB: M. Ghil (ENS, Paris, P.I.),
  S. Hallegatte (CIRED), B. Malamud (KCL, London), A. Soloviev (MITPAN,Moscow),
  P. Yiou (LSCE, Gif s/Yvette, Co-P.I.)